Application of Decomposition Analyses and Sobol Sensitivity Measures to Yield per Recruit and Spawning Stock Biomass per Recruit for several Northeast Species

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Introduction

Stock assessments are complex distillations of commercial and recreational catches, fishery independent data, and biological information into estimates of current stock size, rates of removal, and biological reference points. Such distillations are used to craft scientific advice on fisheries management but are less frequently used to identify ecosystem changes affecting stock status. Changes in rates of growth, maturation and natural mortality can be viewed as the integration of one or more ecosystem factors. In this paper we introduce several approaches that can be used to decompose the overall changes in biological reference points into the differences induced by changes in growth, maturity, and natural and fishing mortality rates. Identification of the timing and magnitude of such changes may be useful for future ecosystem analyses.

One of the foundations of biological reference points in fisheries science is the concept of expected yield per recruit (YPR) and spawning biomass per recruit (SBPR). YPR, the total expected catch in weight from an individual over its lifetime, can be written as a complex function of fishing mortality, natural mortality, and average weight at age. Yield at any given age is derived from the Baranov catch equation $Y_a=F_a/Z_a$ (1-exp($-Z_a$) N_a W_a where F_a is fishing mortality at age a, Z_a is total mortality rate (F_a+M_a) where M_a is natural mortality, W_a is the average weight at age a, and N_a is the expected fraction of recruits alive at age a. When $N_1=1$, the YPR=ΣY_a is the lifetime yield from an individual alive at age 1. YPR is thus an integral quantity that reflects the composite effects of growth, natural mortality, and fishing mortality. Fishing mortality can be controlled by fisheries managers in a variety of ways which leads to concept that the productivity of a wild fishery resource can be specified by setting harvest or effort limits. The total expected yield from a resource is simply the product of the YPR and the average number of recruits (YPR * Rbar). The concept of YPR can be extended to consider spawning stock biomass per recruit (SBPR) which defines the expected reproductive potential of a recruit over its lifespan.

Together, YPR and SBPR can be used to define biological reference points for fisheries. In particular, defining an optimal level of SBPR allows one to find a fishing mortality rate sufficient to ensure some fraction of the maximum SBPR that occurs when F=0. When annual recruitment varies about some mean value the maximum sustainable yield (MSY) is simply the product of YPR at the optimal fishing mortality rate F_{msy} proxy and average recruitment. Similarly, the optimal spawning stock biomass (SSB) is the product of SBPR at the optimal F and average recruitment.

YPR and SBPR models are therefore important methods for integrating information about stock condition and defining harvest strategies. As stock assessments are updated over time, changes in biological parameters and intensity of fishing mortality result in updated values of MSY and SSBmsy. Such changes also reflect changes in the ecosystem. For example, how is MSY affected by a reduction in average weights at age? How does a change in selectivity affect SSBmsy? What is the joint effect of a reduction in weights at age and shift in fishery selectivity towards older fish? Identification of timing and magnitude of such changes could be of assistance in the identification of causal mechanism (eg., change in plankton productivity, increases in temperature, or availability of forage species). Hence, there is a strong connection between the changes summarized in stock assessments with changes observed in the ecosystem.

Dynamic changes in YPR and SBPR between two time periods occur when the input parameters change in response to harvesting and natural factors. We define those input parameters $\boldsymbol{\Theta}$ as the set of age-specific fishing mortality **F**, natural mortality **M**, weights **W**, and maturity **Mat** where $\boldsymbol{\Theta} = \{\mathbf{F}, \mathbf{M}, \mathbf{W}, \mathbf{Mat}\}$. Keyfitz (1967, 1968a, 1968b) first introduced an approach to examine the consequences of a shift in parameters at time t1 ($\boldsymbol{\Theta}_{t1}$) to a new state at time t2 ($\boldsymbol{\Theta}_{t2}$). We demonstrate that the Keyfitz method is based on an approximation of the total differential of YPR and SBPR. A consequence of this approximation is a quantity variously called an "interaction effect" or "residual" in the literature.

We then extend the Keyfitz approach using a method from the human demography literature (Horiuchi et al. 2008). Their method computes total effect of parameter changes from two points in time on the total differential. The Horiuchi method uses a line integral for each parameter change from $\boldsymbol{\Theta}_{\mathbf{t}}$ to $\boldsymbol{\Theta}_{\mathbf{t+1}}$. The Horiuchi method eliminates the need for estimation of an interaction effect because the true rates of change along the path between time t and time t+1 are more closely approximated by controlling the parameterization of the integration. The Horiuchi method not only gives an exact solution to the total differential, but also eliminates the need for an interaction term.

Finally, we apply methods of sensitivity analyses based on Sobol (1993) and Puy et al (2022) that can provide additional insights into important parameters for monitoring, and/or critical assumptions. Using the Sobol method, we then examine develop empirical measures of system performance (total yield in a given year, estimated spawning stock biomass) based on controllable (**F**) and uncontrollable (**M, W, Mat,** $\mathbf{R_t}$**)** factors.

All of the above methods are applied to recent stock assessment results for Georges Bank haddock, bluefish, and summer flounder.

Methods

The Basics

For the purposes of this paper, we consider YPR and SBPR as scalar functions of four vectors **Θ**={**F, M, W, Mat**} where each vector has A elements corresponding to the age = 1, 2, …A. For the sake of notational simplicity, it is assumed that the cohort is recruited at age 1. Without loss of generality, it is assumed that the force of mortality at age can be written as the product of age specific selectivities S_a multiplied by the scalar F_{full}. In our notation the vector $\mathbf{F}=F_{full}\mathbf{S}$ where **S** is the vector of selectivities S_a, a=1,…A. For overfished status, one can use the same parameters **Θ**={**F, M, W, Mat**} to compare the observed SSB to the predicted SSB given the realized set of recruitments $\mathbf{R_t}$={R_{t-1}, R_{t-2}, R_{t-3}., ..}.

Data necessary to compute relative Yield and relative SSB are available for all age-based assessments. These variables are defined below:

$\mathbf{Y_t}$= observed yield in year t where yield in the sum of landings $\mathbf{L_t}$ and discards $\mathbf{D_t}$

$\mathbf{R_t}$ = observed recruitment in year t.

M=natural mortality, generally assumed to be constant at age.

$\mathbf{F_{t,a}}$=vector of age specific mortality rates in year t where a=1,2,…A, where **A**= maximum age. $\mathbf{F_{t,a}}$ is often written as **F** $\mathbf{S_{t,a}}$ where $\mathbf{S_{t,a}}$ is the selectivity at age.

$\mathbf{F_{opt,a}}$= **Fmsy** * $\mathbf{S_a}$ where **Fmsy** is some proxy for MSY, and **Sa** is the age specific selectivity

$\mathbf{G_{t,a}}$= probability of surviving to age a in year t

$\mathbf{W_{t,a}}$= average weight of fish of age a in year t.

$\mathbf{Mat_{t,a}}$= fraction of age a that is mature in year t

The expected yield in year t from a cohort of age a is a function of the initial cohort size $\mathbf{R_t}$, the probability of living to age a ($\mathbf{G_{t,a}}$), the fishing mortality rate at age a, and the average weight $\mathbf{W_{t,a}}$. The expected total yield in year t is simply the sum over all cohorts that constitute the extant population {R_{t-1}, R_{t-2}, …R_{t-A}}

$$E(Y_t) = \sum_{a=1}^{A} G_{t,a-1} \frac{F_{t,a}}{F_{t,a}+M}\left(1-e^{-(F_{t,a}+M)}\right) R_{t-a} W_a$$

The variable $\mathbf{G_{t,a}}$ is simply the probability of living to age a and is equivalent to the $\mathbf{l_x}$ term in the Euler-Lotka equation. In fisheries notation it is

$$G_{t,a} = e^{-\sum_{i=1}^{a} F_{t,i}+M}$$

If all R_t equal a constant value R for all years, then $E(Y_t)/R$ is simply equal to yield per recruit at equilibrium.

$$YPR = \frac{Y}{R} = \sum_{a=1}^{A} \frac{F_{t,a}}{F_{t,a} + M} \left(1 - e^{-(F_{t,a}+M}\right) e^{-\sum_{i=1}^{a-1} F_{t,i}+M} W_a$$

A similar exercise can be used to compute the expected SSB in year t given fishing mortality rate and selectivity pattern **F** and biological parameters **W**, **M,** and **Mat**.

$$E(SSB_t) = \sum_{a=1}^{A} G_{t,a-1}\, R_{t-a} Mat_{t,a} W_a$$

As with YPR, if all R_t equal a constant value then $E(SSB_t)/R$ is simply equal to spawning stock biomass per recruit at equilibrium.

$$SBPR = \frac{SSB}{R} = \sum_{a=1}^{A} e^{-\sum_{i=1}^{a-1} F_{t,i}+M}\, Mat_{t,a} W_a$$

SBPR decreases monotonically as F increases.

At equilibrium, the expected yield is defined as R * YPR, and expected Spawning Stock Biomass is R * SBPR.

Reference points for a stock can be defined from perspective of some function of yield maximization or conservation of sufficient spawning stock per recruit that is expected to produce "optimal" long term recruitment, or more pessimistically reduce the risk of stock collapse. Much of the stock assessment literature for the past 60 years has addressed these issues. Assuming one "knows" the fraction of potential SBPR that is appropriate for a given set of objectives, a fishing mortality rate can be estimated to achieve that fraction. This type of reference point is usually defined as a $F_{x\%SPR}$. As the x%SPR value is an approximation of the true value for maximum sustainable yield, the $F_{x\%SPR}$ is often called a proxy value. Thus the proxy for maximum sustainable yield is defined as

$$MSY = R * YPR(F_{x\%SPR}).$$

The proxy value for biomass at MSY is simply

$$SSB_{msy} = R * SBPR(F_{x\%SPR}).$$

Total Differential for MSY and SSB_{msy}

MSY and SSBmsy are complex scalar functions of R and the parameter vector **Θ**. The total differential concept can be used to illustrate the changes induced by over time. By definition, the total differential for MSY and SSBmsy can be written as:

$$\Delta MSY=(R+\Delta R)\ (YPR_{msy} + \Delta YPR_{msy}) - R * YPR_{msy}$$

$$\Delta SSB_{msy} = (R + \Delta R)(SBPR_{msy} + \Delta SBPR_{msy}) - R * SSB_{msy}$$

The effect of a change in recruitment is proportional to ΔR/R, and the effect of change in YPR or SBPR is simply ΔYPR/YPRmsy or ΔSBPRmsy/SBPRmsy, respectively. The effect of changes in R is simple to compute, but understanding the effects of changes in YPR or SBPR is complicated by the number of parameters and their varying rates of change over time. As demonstrated in a later section, the total differential is function of partial derivatives of MSY and SSBmsy with respect to each parameter and the rate of change of those parameters over time.

Moreover, the changes in parameters occur over the time interval between assessments. Can we disaggegate those effects to better understand the processes giving rise to the changes? Can we decompose the total differential into the effects of these changes? The simple answer is yes. First, we begin by adapting approaches that have been used in demography for life table functions.

Decomposition Analyses

Keyfitz: First Order Effects

We illustrate some potentially useful ways of describing the total differential for YPR and SBPR as a function of the vector changes in **F, M, Mat**, and **W**. In the late 1960's Keyfitz (1967, 1968a, 1968) proposed an approach to decompose they rate of change of demographic functions (e.g., average age in a population) into changes due to survival and fertility. Keyfitz expressed the change in average age as the difference in the functions with the baseline and changed vectors of survival and fertility. Herein we apply the same principle to define the joint effect of changes in YRP (Eq. yy) and SBPR (Eq. xx). We use the general notation **Θ**={**F, M, W, Mat**} to define the baseline parameters for the functions YPR() and SBPR(), such that YPR(**Θ**) and SBPR(**Θ**) define the baseline estimates of YPR and SBPR respectively. Let **Θ' =**{**F', M', W', Mat'**} represent the change in parameters. The joint effects of changes in all four vectors is expressed as

$$_{s}\delta_{T} = SBPR(\boldsymbol{\Theta}')-SBPR(\boldsymbol{\Theta})$$

$$_{s}\delta_{T} = SBPR(\mathbf{W', F', Mat', M'}) - SBPR(\mathbf{W, F, Mat, M})$$

The total difference for YPR(Θ) can be written as

$$_{y}\delta_{T} = YPR(\boldsymbol{\Theta}')-YPR(\boldsymbol{\Theta})$$

$$_{y}\delta_{T} = YPR(\mathbf{W', F', Mat', M'}) - YPR(\mathbf{W, F, Mat, M})$$

For notational simplicity we will drop the s and y subscripts on δ in the following description. The full notation for the YPR and SBPR models may be found in Appendix 1.

Keyfitz noted that the total difference could be decomposed into component vectors for survival and fertility by evaluating the expected average age function for changes in vectors. In other words, the changes for a similar parameter type are made all at once. Applying this same principle to YPR, we obtain.

$$\delta_W = YPR(W', F, Mat, M) - YPR(W, F, Mat, M)$$

$$\delta_F = YPR(W, F', Mat, M) - YPR(W, F, Mat, M)$$

$$\delta_{Mat} = YPR(W, F, Mat', M) - YPR(W, F, Mat, M)$$

$$\delta_M = YPR(W, F, Mat, M') - YPR(W, F, Mat, M)$$

Since $\mathbf{F}=F_{full}\mathbf{Sel}$, the differential for $\mathbf{F}$ can be written in terms of F_{full} and Sel as follows:

$$\delta_{Sel} = YPR(\mathbf{W}, F_{full}, \mathbf{Sel'}, \mathbf{Mat}, \mathbf{M}) - YPR(\mathbf{W}, F_{full}\,\mathbf{Sel}, \mathbf{Mat}, \mathbf{M})$$

And

$$\delta_{Ffull} = YPR(\mathbf{W}, F'_{full}, \mathbf{Sel}, \mathbf{Mat}, \mathbf{M}) - YPR(\mathbf{W}, F_{full}\,\mathbf{Sel}, \mathbf{Mat}, \mathbf{M})$$

Note that the above two equations illustrate the decomposition of the F_{full} and Selectivity. This will be particularly important when an updated assessment results in the simultaneous adjustment of both F_{full} and selectivity. In such cases, a focus on simply the rate of change in Fmsy proxy can be misleading.

These can be defined as first order effects. The total effect can be written as the sum of the first order effects plus an interaction term.

$$\delta_T = \delta_W + \delta_F + \delta_{Mat} + \delta_M + \xi_T$$

Keyfitz (1968) and others since then have described the residual ξ_T as an interaction term that follows from the overall nonlinearity of the function. As we will see later, the model is indeed nonlinear, but the interaction effect is an approximation error rather than a direct consequence of the interactions among variables. Without loss of generality, we can use the term “interaction” to mean the overall approximation error. Rearranging terms, the interaction effect can be expressed as

$$\xi_T = \delta_T - (\delta_W + \delta_F + \delta_{Mat} + \delta_M)$$

The magnitude of the interactive term is an expression of the nonlinearity of the YPR and SBPR functions in the vicinity of $\mathbf{\Theta}$ and $\mathbf{\Theta'}$. Equation **zz** can be considered as the total differential of YPR. Applying the chain rule, the total differential can be approximated as the sum of partial differentialsYPR$_\theta$, evaluated at $\mathbf{\Theta}$ such that

$${}_y\delta_T = YPR(\mathbf{\Theta}') - YPR(\mathbf{\Theta}) \sim YPR_W(\mathbf{\Theta})\, d\mathbf{W} + YPR_F(\mathbf{\Theta})\, d\mathbf{F} + YPR_{Mat}(\mathbf{\Theta})\, d\mathbf{Mat} + YPR_M(\mathbf{\Theta}) d\mathbf{M}$$

Note that in the above expression, the vector notation implies a summation of all A elements in each vector **W, F, Mat,** and **M**. The approximation for the total differential is exact when f(**Θ**) is linear in all variables. When f(**Θ**) is nonlinear in one or more variables, the approximation becomes exact as **Θ–Θ'** approaches zero, i.e., dM→0, dW→0, dF→0, dMat→0. These properties of the total differential will be considered later when we explore the overall model sensitivity analyses and the additivity of the δ components.

Keyfitz: Second Order Effects

If the function is linear, or nearly linear in the vicinity of Q, then the joint effect of two or more changes can be expressed as the sum of the first order effects. As an illustration, consider the joint effect of changes in **W** and **F**.

$$\delta_{WF} = YPR(W', F', Mat, M) - YPR(W, F, Mat, M)$$

Which can be approximated as

$$\delta_{WF} = \delta_W + \delta_F + \xi_{WF}$$

In general, ξ_{WF} will be zero when the variables affect the function linearly.

All of the equations above are implemented in R. The YPR and SBPR coding relies on the ypr and sbpr functions in the R package "fishmethods" written by Gary Nelson, MADMF.

Derivation of Horiuchi Model for YPR and SBPR

The "interaction effect" described by Keyfitz has been variously described as a consequence of the nonlinearity of the model or the mixture of linear and nonlinear components (Das Gupta + others) . A true interactive effect, as commonly conceived in analyses of variance or general linear models, is probably not applicable. Slower growth may lead to reduced selectivity at age if fishing effort does not shift to areas and times when smaller fish are available. Similarly, delayed growth in total weight can sometimes be a function of earlier maturation or it may simply reflect the survival of greater fraction of smaller fish due to fishing mortality. Identification of true causes, however, is not generally possible without additional experimental evidence.

Horiuchi et al. (2008) first noted that the "interaction effect" is just an approximation error associated with examining the total differential that does not consider the effect of time. Their methodology demonstrated for the first time that the additivity of parameter effects was true even if the function was nonlinear. Previous methods (Keyfitz, Gupta etc) had not explicitly examined the effects of changes in the parameters over time nor had some basic rules of calculus been applied. In the methods below we apply Horiuchi's method to the YPR problem. The notation for SBPR can be derived in parallel.

First, let **Θ** denote the vector quantity $[\theta_1, \theta_2, \ldots \theta_N]$ where the individual qs map to each of the A elements of the vectors **W, F, Mat,** and **M.** Thus N is equal to 4*A. For each parameter qi, denote its value at t_1 as $\theta_i(t_1)$ and value at t_2 as $\theta_i(t_2)$.

The total difference in YPR between time periods is thus

$$\text{YPR}(t_2) - \text{YPR}(t_1) = \int_{t_1}^{t_2} \frac{d}{dt}\, YPR(t)\, dt$$

The partial derivative for YPR with respect to the parameter θi is denoted as

$$YPR_{\theta_i} = \frac{\partial}{\partial \theta_i(t)} YPR(\Theta, t)$$

Applying the chain rule for partial derivatives of a composite function, results in

$$\text{YPR}(t_2) - \text{YPR}(t_1) = \int_{t_1}^{t_2} \sum_{i=1}^{N} \left\{ \frac{\partial}{\partial \theta_i(t)} YPR(\Theta, t)\ \frac{d}{dt}\ \ \theta_i(t) \right\} dt$$

Applying the substitution rule the integral and summation operators can be reversed such that

$$\text{YPR}(t_2) - \text{YPR}(t_1) = \sum_{i=1}^{N} \int_{\theta_i(t_1)}^{\theta_i(t_2)} \left\{ \frac{\partial}{\partial \theta_i(t)} YPR(\Theta, t)\, d\theta_i(t) \right\}$$

The term within the integral on the right hand side can be denoted as ci

$$\text{YPR}(t_2) - \text{YPR}(t_1) = \sum_{i=1}^{N} c_i$$

Thus c_i is the change produced by the change in θ_i from time 1 to time 2.   Following Horiuchi et al., the above equation defines a line intergral. The numerical approximation of the c_i was obtained by integrating over n=100 steps between t_1 and t_2 using the R package DemoDecomp. The Horiuchi method relies on approximately continuous changes in the θ_i rather than a discrete change as in the Keyfitz and related methods. A particularly useful feature of this additivity property is that one can assess the composite change in YPR induced by the vector change in say, **W**(t_1) to **W**(t_2) by summing the ci corresponding to the elements in θ_i. Similarly, it is easy to separate the effects of changing in selectivity from the effects of changes in full F.

The R package DemoDecomp was used to compute the Horiuchi function for YPR and SBPR.

Sensitivity Analyses: Application of Sobol Method

The decomposition methods of Keyfitz and Horiuchi provide insights into the effects of model parameters on the total rate of change in MSY and SSBmsy.   However, they do not provide direct information on effect of parameter uncertainty on the estimates. To explore this aspect of MSY and SSBmsy we used the method of Sobol(1967), as implemented in the R package sensobol, described by Puy et al. 2022a. These authors emphasize the utility of so called "global sensitivity" methods using variance-based estimators of first and all higher order interactions.

Essentially, the Sobol methods are designed to fully explore the N-dimensional parameter space for complex functions. Puy et al. (2022b) define a total order index T_i for each parameter θ_i, i=1,2,…N such that the $\Sigma T_i=1$. The variance estimator is derived by first constructing a J by 2N matrix Q where the columns represent the parameters and J is he number of sample points evaluated. For notational convenience the matrix Q can be considered as two submatrices A and B, both to which are J x N matrices. Denote the matrix $A_B^{(i)}$ as a matrix with all the columns from A except the i^{th} which comes from B.

For notational simplicity let y= f() represent either MSY() or SSBmsy(). The total variance of y is defined as

$$Var(y) = \frac{1}{2J-1} = \sum_{j=1}^{J}\left(\left(f(A)\right)_j - f_0\right)^2 + \left(\left(f(B)\right)_j - f_0\right)^2\right)$$

Where

$$f_0 = \frac{1}{2J} = \sum_{j=1}^{J} f(A)_j + f(B)_j$$

Based on a comparison of multiple estimation methods, we used the Jansen(1999) total order estimator which is defined as

$$T_i = \frac{\sum_{j=1}^{J}\left(f(A)_j - f(A_B^i)\right)\left(f(A)_j - f(A_B^i)\right)}{Var(y)}$$

The samples are selected using a quasi-random number generator in which a matrix column.

The R package sensobol is used for all computations of Sobol Indices.

Application of Keyfiz, Horiuchi and Sobol Methods to Georges Bank haddock, Bluefish, and Summer Flounder

The Keyfitz and Horiuchi methods focus on the effects of one or more input parameters on the overall difference between two estimates biological reference points at different points in time. They both follow from an evaluation of the total differential but the Horiuchi method is a numerical approximation of the exact solution. The earlier Keyfitz method has a stronger intuitive appeal but the failure to consider the trajectory of parameter changes over time results in an "interaction" or "residual" term. The interaction term has been variously described as a consequence of the nonlinearity of the function or joint effects of parameter changes.

Changes in BRPs can arise due to changes in the

1. Pattern of fishing,
2. Environmental or density-dependent effects on biological processes, and
3. Changes in the assessment model used to evaluate stock status.

These changes can interact. For example, reductions in growth rates may shift fishery selectivity to older age classes. Changes in the assessment model structure or new data sources can lead to alternative understandings of stock scale or biological processes.

The Sobol method provides a way evaluating the importance of each parameter on the overall uncertainty of the estimate in the neighborhood of the solution. Advantage of this approach compared to other Monte Carlo approaches include direct use of parameter uncertainty in the parameter improved likelihood of exploring the parameter space. The Sobol method allows for partitioning to the total uncertainty into the contributions from each parameter.

The Horiuchi and Sobol method both have useful property of additivity. The sum of Horiuchi indices is equal to the total differential. The sum of the Sobol total order indices is one; hence each index can be viewed as the fraction of total variance attributable to the parameter. In turn, this property can be used to evaluate the totals attributable to say overall changes in weights at age, maturity and so forth.

Three case studies were selected to illustrate the properties of these methods. Georges Bank haddock illustrates the changes in reference points driven by reductions in weights at age over a 12-yr period. Bluefish illustrates the effects of a major change in an assessment model and improved understanding of natural mortality. Summer flounder illustrates the effects of several small changes in model parameterization.

Georges Bank Haddock

Georges Bank haddock comparisons between 2005 and 2017 (Table 1) revealed major reductions in weight at age W and a shift in selectivity towards older fish. Maturity of 2 and 3-year old fish declined slightly in 2021 and M was assumed to be 0.2 for both assessments. The composite effect of these changes (Table 8) was a 2.1-fold increase in F35%SPR from but a 45% decline in SBP and 39% decline in YPR (from 0.607kg/R to 0.372 kg/R). Nearly all of the change was due to change in weight, despite the nearly two-fold increase in F%MSP. SBPR declined by 45% from 2.89 to 1.60 kg/R. The large increase in Fmsp was offset almost entirely by the change in selectivity. A 2.1 fold increase in Fmsp would have increased SSBmsy by 1.13 kg/R if the selectivity pattern had not shifted to older fish. Selectivity pattern change alone decreased SSBmsy by 1.13 such that the composite effect of Fmsp and Selectivity resulted in a change of only 0.008 kg/R. The force of mortality is more appropriately characterized by the total change in **F**, defined as F_{full}***Sel.** The slight decline in maturation for ages 2 and 3 induced a 0.14 kg/R change in SBPR but had no effect on YPR (by definition, since YPR is not a function of maturation).

One of the important differences between the Keyfitz and Horiuchi methods is the elimination of the interaction effect in the Horiuchi method. The Keyfitz method estimated relatively large interaction effect of -7% and -12% for SBPR and YPR respectively (Table 2). The Horiuchi effects, both at the aggregate (Table 2) and age specific level (Table 3) are somewhat lower than the Keyfitz method. The second order effects (Table 2) generally show good agreement in terns of summation of the main effects, but the sum of the **W** and **F** effects differed by about 5% (e.g., δ_{WF}=-0.23558 whereas $\delta_W+\delta_F$=-0.25326+0.031193=-0.2221). It is important to note that the sum of Keyfitz first order effects plus the interaction effect is equal to the total difference. Similarly, the sum of the Horiuchi indices is exactly equal to the total differential.

The Sobol Total Order indices provide a useful way of characterizing the effects of uncertainty in the input parameters on the total uncertainty of the estimate. In this illustration, the input parameters were all assumed to uniformly distributed about the mean with a range of +10% and -10% for the upper and lower bounds, respectively. The overall effects of such variability on SBPR and YPR is illustrated for the baseline and new parameter estimates (Fig. 4). A more realistic monte carlo exercise would consider actual measures of variation and alternative probability density functions. An interesting aspect of the Sobol total order estimates is that they are specific to the neighborhood of the parameterization. Figure 5 illustrates the differential effects of variation in W in 2005 and the lighter average weight fish in 2017.

TABLE 1. Georges Bank Haddock input tables from 2005 to 2017. Input from the 2008 assessment was not considered in this report.

Table 4a. Summary of input data for GB haddock based on Garm II 2005

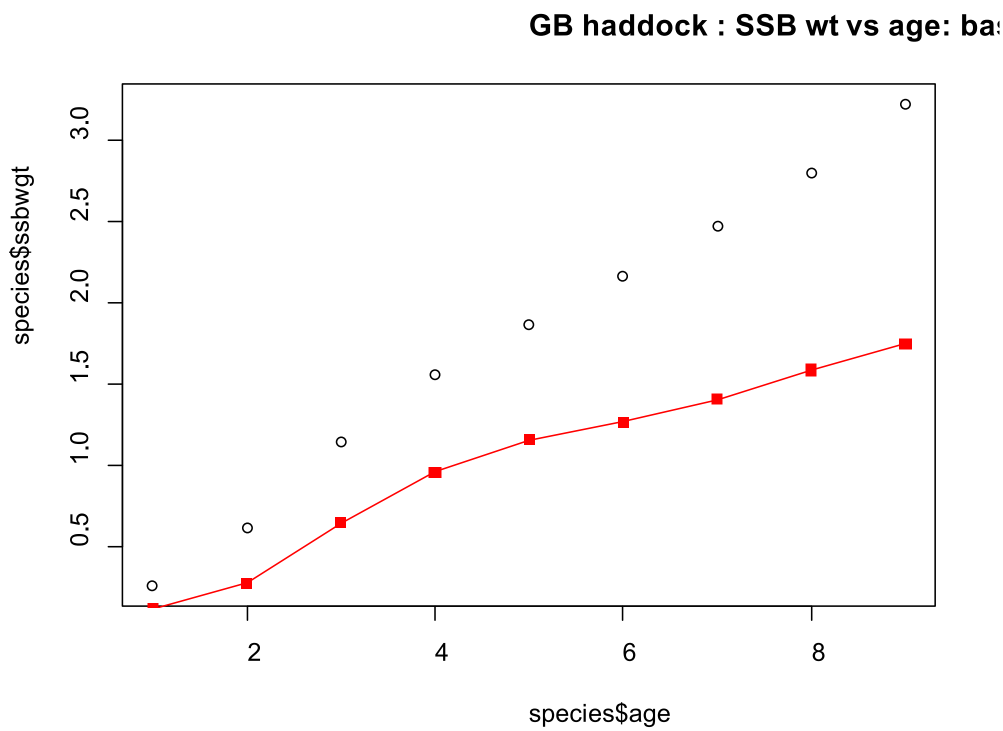
age	*ssbwgt*	*partial*	*pmat*	*M*
1	0.26	0	0.01	0.2
2	0.62	0.09	0.55	0.2
3	1.15	0.47	0.95	0.2
4	1.56	0.92	0.99	0.2
5	1.87	1	1	0.2
6	2.17	1	1	0.2
7	2.48	1	1	0.2
8	2.8	1	1	0.2
9	3.23	1	1	0.2

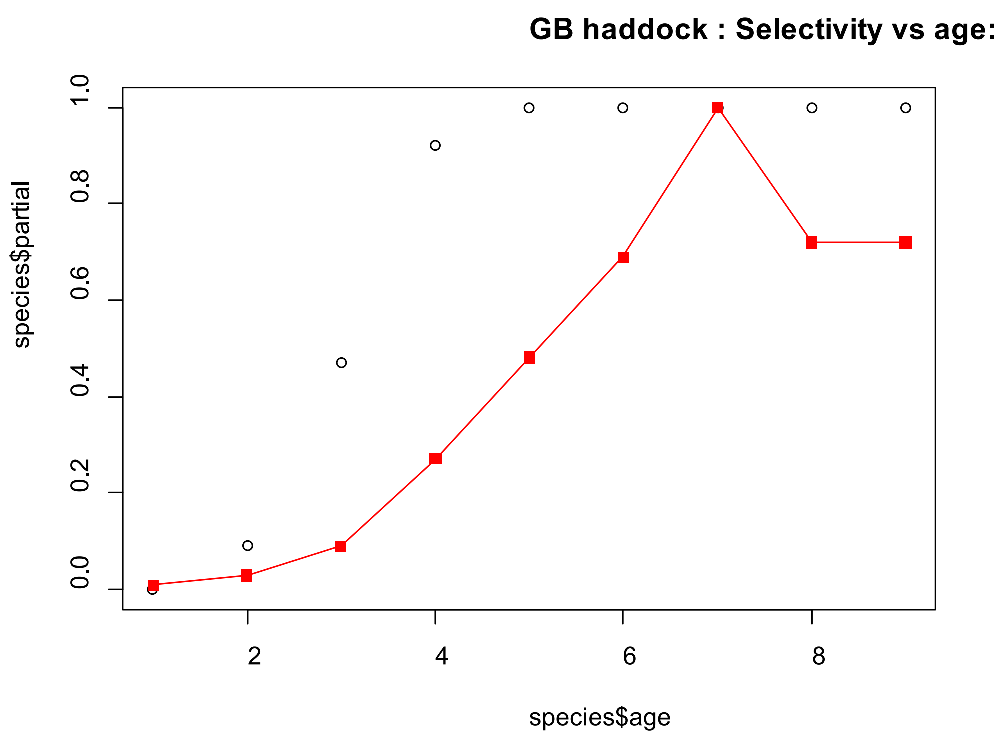
Table 4b. Summary of input data for GB haddock based on Garm III 2008

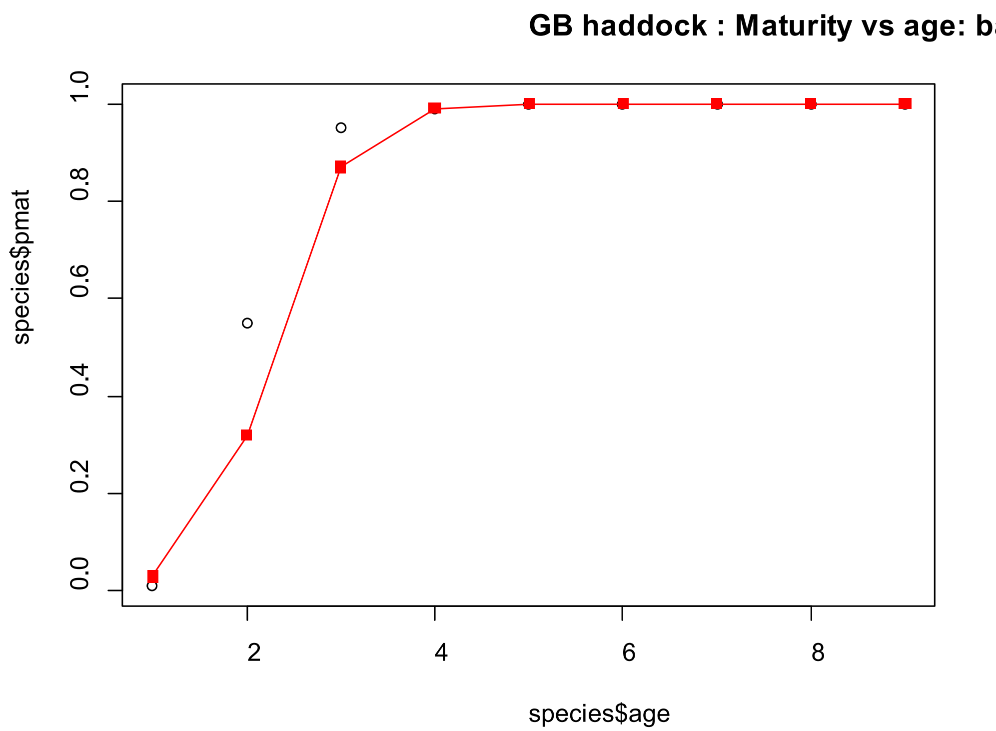
age	*ssbwgt*	*partial*	*pmat*	*M*
1	0.11	0.01	0.06	0.2
2	0.36	0.03	0.47	0.2
3	0.8	0.15	0.92	0.2
4	1.25	0.4	0.99	0.2
5	1.56	1	1	0.2
6	1.82	1	1	0.2
7	2.05	1	1	0.2
8	2.34	1	1	0.2
9	2.64	1	1	0.2

Table 4c. Summary of input data for GB haddock based on 2017 Update assessment

age	*ssbwgt*	*partial*	*pmat*	*M*
1	0.12	0.01	0.03	0.2
2	0.28	0.03	0.32	0.2
3	0.65	0.09	0.87	0.2
4	0.96	0.27	0.99	0.2
5	1.16	0.48	1	0.2
6	1.27	0.69	1	0.2
7	1.41	1	1	0.2
8	1.59	0.72	1	0.2
9	1.75	0.72	1	0.2


GB haddock : SSB wt vs age: ba
species$ssbwgt
0.5
1.0
1.5
2.0
2.5
3.0
2
4
6
8
species$age


GB haddock : Selectivity vs age:
species$partial
0.0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
species$age



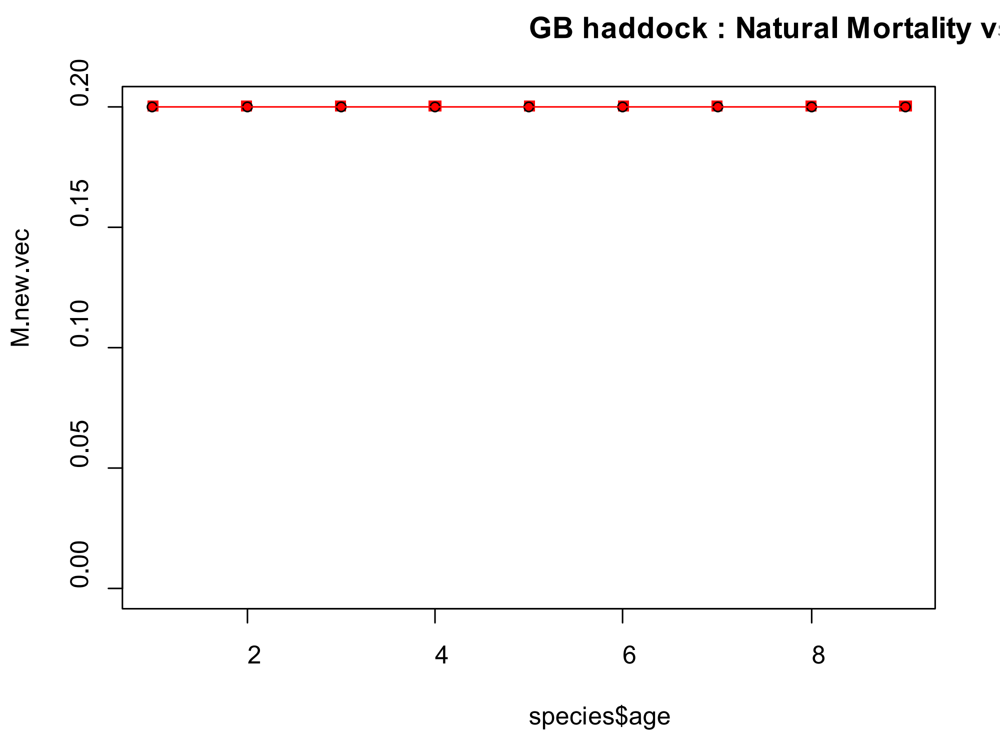


Figure 1. Comparison of changes in weight at age, selectivity at age, maturation at age, and natural mortality at age for Georges Bank haddock between 2005 (black open dots) and 2017 (red closed squares).

Table 2. Decomposition of YPR and SBPR functions for Georges Bank haddock using the Keyfitz (left columns) and Horiuchi methods (right columns). Input data are from 2005 and 2017 assessments in Table 1. The age-specific Horiuchi effects are presented in Table 3.

Keyfitz and Horiuchi Model Estimates of composite effects of Weight, Selectivity, maturation and M on SSB and YPR.

	KEYFITZ DELTA MODEL							HORIUCHI MODEL			
Factor	Fmsp	SSBmsp	YPRmsp					SSBmsp	YPRmsp		
Base Model	0.222536	2.891893	0.607844					2.891893	0.607844		
New Model	0.442816	1.603274	0.372267					1.603274	0.372267		
New - Base	0.22028	-1.28862	-0.23558	Percent of total dif				-1.28862	-0.23558	Percent of total dif	
First Order Effects				SSBmsp	YPRmsp					SSBmsp	YPRmsp
W		-1.23516	-0.25326	96%	108%			-1.13468	-0.26402	88%	112%
F		-1.12704	0.093166	87%	-40%			-0.95022	0.096211	74%	-41%
Sel		1.127181	-0.10341	-87%	44%			0.900778	-0.06777	-70%	29%
F* Sel		0.0081	0.031193	-1%	-13%			-0.04944	0.028439	4%	-12%
Mat		-0.1398	0	11%	0%			-0.1045	0	8%	0%
M		0	0	0%	0%			0	0	0%	0%
Interaction 1		0.086204	0.027932	-7%	-12%					NA	NA
CHECK SUM		-8.8E-07	-4E-08					-8E-07	-2E-08		
2nd Order Effects		SSBmsp	YPRmsp	Pred SSB	% dif	Pred YPR	% dif				
W*F* Sel		-1.2186	-0.23558	-1.23502	-1.3%	-0.26351	-11.9%				
W* Mat		-1.30359	-0.25326	-1.3749662	-5.5%	-0.25326	0.0%				
F* Sel *Mat		-0.13454	0.031193	-0.131701822	2.1%	0.031193	0.0%				
W* M		-1.23516	-0.25326	-1.235164	0.0%	-0.25326	0.0%				
M* Mat		-0.1398	0	-0.1398022	0.0%	0	0.0%				
F* Sel * M		0.0081	0.031193	0.008100378	0.0%	0.031193	0.0%				

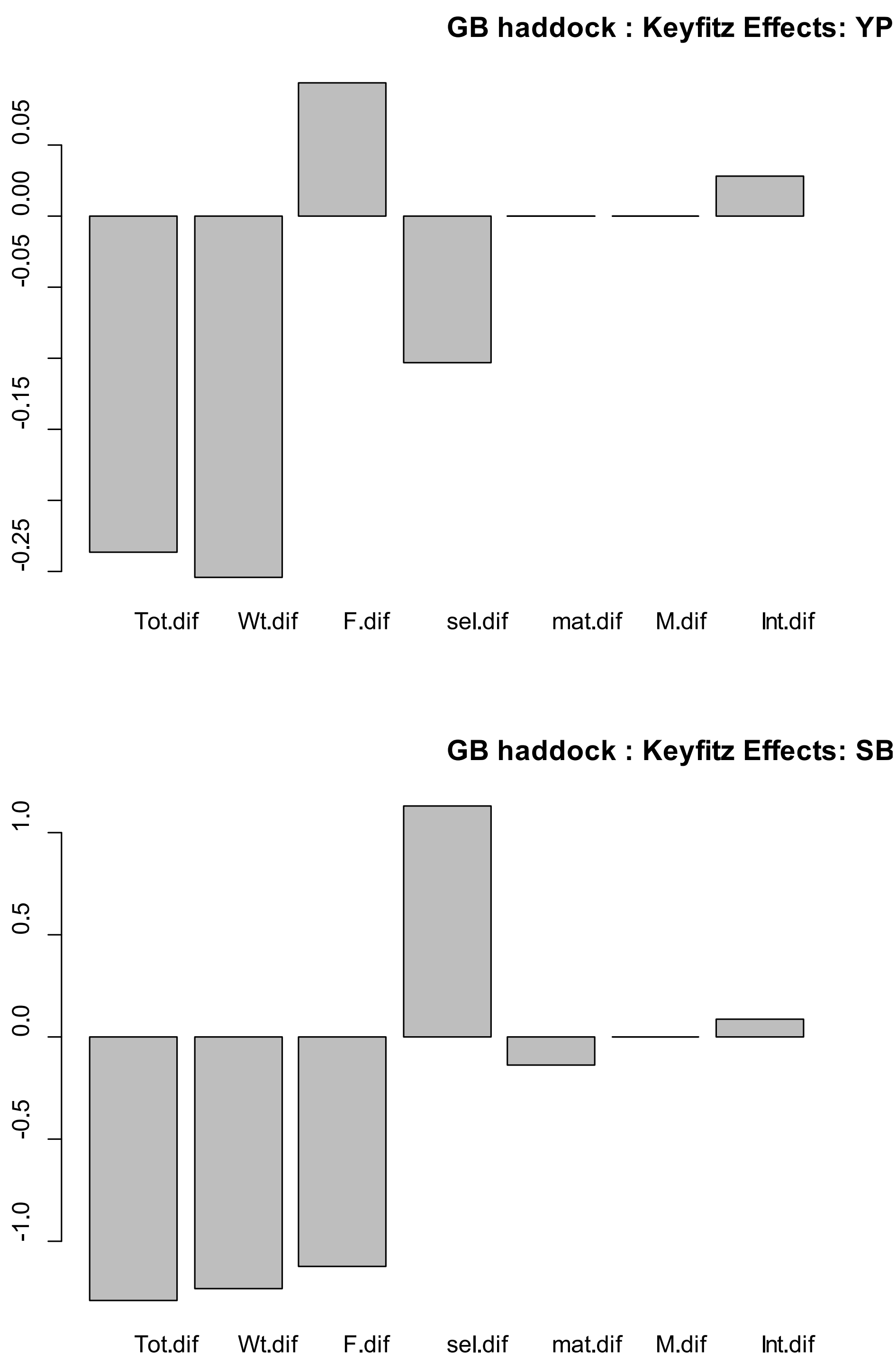


Figure 2. Graphical depiction of the Keyfitz decomposition of YPR (top) and SBPR(bottom) by parameter type for Georges Bank haddock . Units of y-axis are in kg. Results are listed in Table 2.

Table 3. Decomposition of YPR (top) and SBPR (bottom) functions for Georges Bank haddock using the Horiuchi method by variable type and age.

Table xx. Summary of Horiuchi effects for SSB and MSY by Weight, selectivity, maturity and natural Mortality by age for Georges Bank Haddock.					
Variable	***Age***	***Wt.ssb.H***	***Sel.ssb.H***	***pmat.ssb.H***	***M.ssb.H***
SSB	1	-0.00237	-0.00683	0.003214572	0
SSB	2	-0.10078	0.040468	-0.07052718	0
SSB	3	-0.23552	0.233999	-0.037187523	0
SSB	4	-0.21284	0.310511	0	0
SSB	5	-0.1659	0.175643	0	0
SSB	6	-0.13151	0.07085	0	0
SSB	7	-0.09269	0	0	0
SSB	8	-0.06435	0.028141	0	0
SSB	9	-0.12871	0.047991	0	0
SSB	SUM	-1.13468	0.900778	-0.104500131	0
		Ffull	R.ave	pF	pM
SSB		-0.95022	0	0	0
Variable	***Age***	***Wt_Hor***	***Sel_Hor***	***pmat_Hor***	***M_Hor***
MSY	1	-0.00023	-0.00107	0	0
MSY	2	-0.00471	0.00336	0	0
MSY	3	-0.02455	-0.00489	0	0
MSY	4	-0.04547	-0.02442	0	0
MSY	5	-0.04563	-0.01841	0	0
MSY	6	-0.0429	-0.00825	0	0
MSY	7	-0.03714	0	0	0
MSY	8	-0.02125	-0.00445	0	0
MSY	9	-0.04213	-0.00964	0	0
MSY	SUM	-0.26402	-0.06777	0	0
		Ffull	R.ave	pF	pM
MSY		0.096211	0	0	0

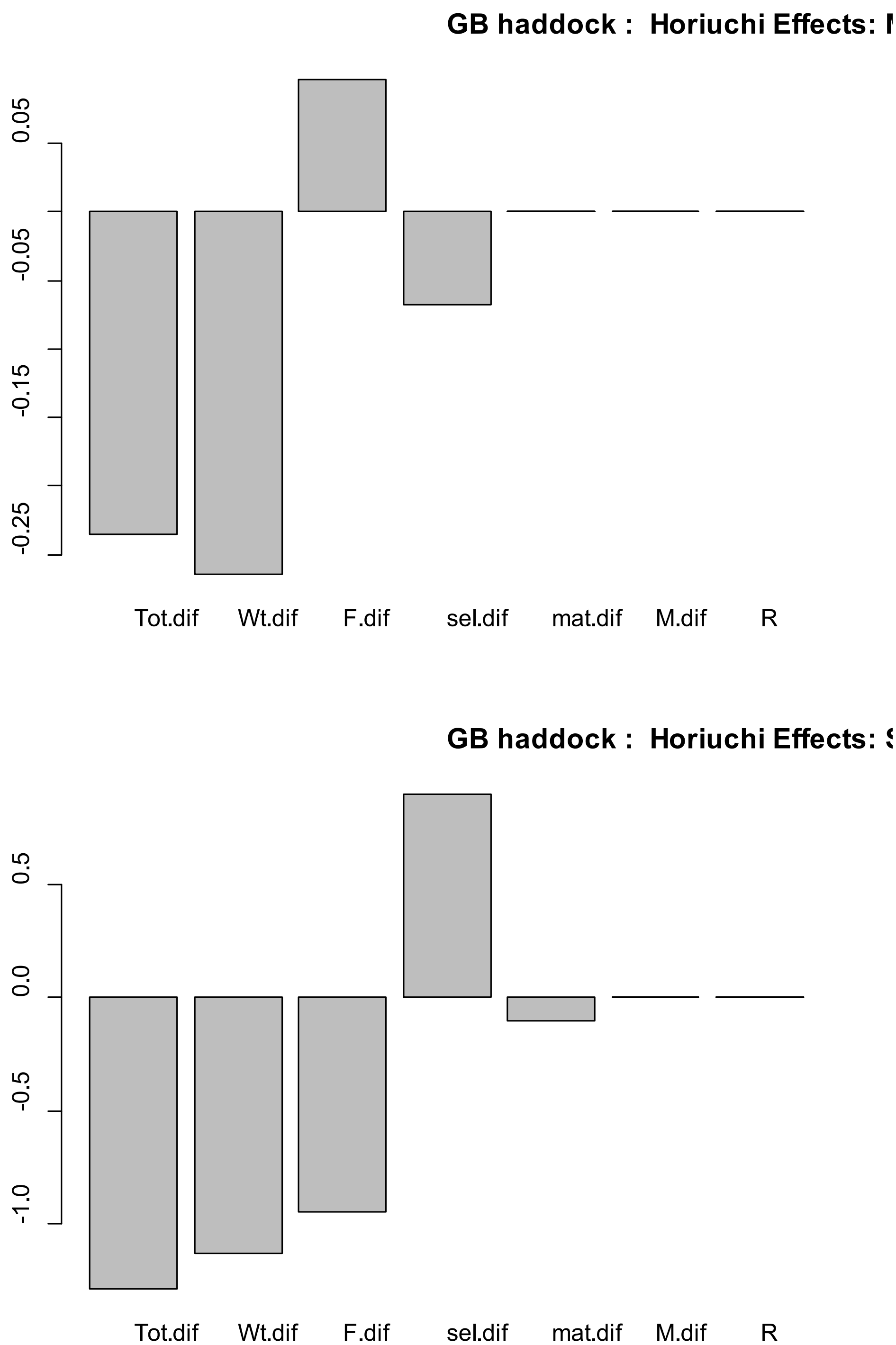


Figure 3. Graphical depiction of the Horiuchi decomposition of YPR (top) and SBPR(bottom) by parameter type for Georges Bank haddock . Units of y-axis are in kg. Results are listed in Table 2.

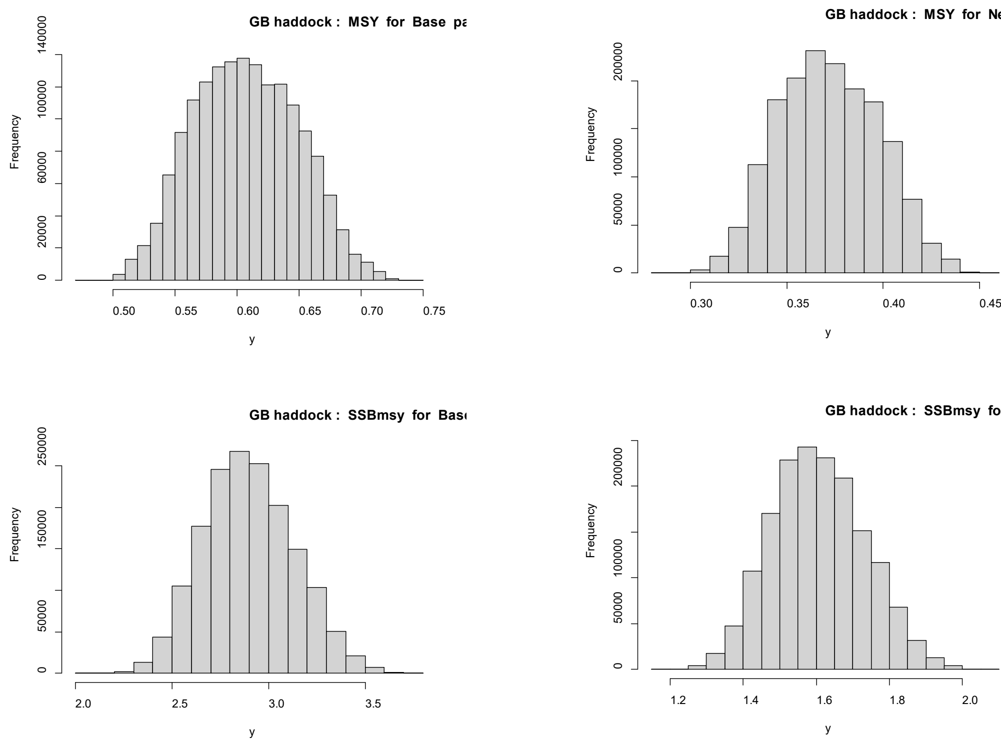


Figure 4. Distribution of MSY and SSBmsy for the baseline and new parameters for Georges Bank haddock based on the Sobol method. Variation of all input parameters by +/- 10%. Average recruitment is assumed to be 1.0 such that the above graphs are equivalent to the YPR and SBPR distributions, Units of x-axes are in kg.

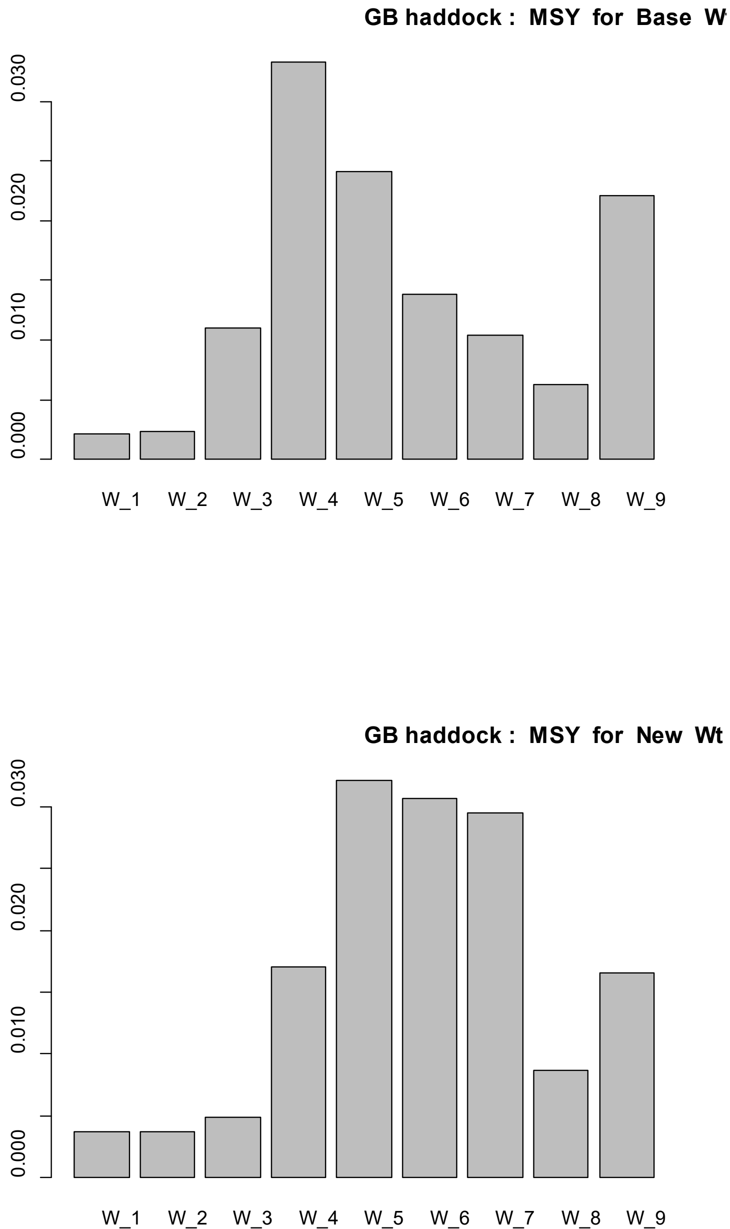


Figure 5. Comparison of the Sobol indices for Weight at age for the baseline (top) and new (bottom) parameter estimates for MSY for Georges Bank haddock. This figure demonstrates the local effect of system state on the Sobol indices; Sobol indices depict the sensitivity of the MSY model in the vicinity of the solution. Age specific estimates for each parameter are listed in Table 4.

Table 4. Summary of age specific Sobol Total Order indices for Georges Bank. Table entries represent the fraction of total variance in MSY or SSBmsy explained by the input variables. Sub tables illustrate the effects for MSY and SSBmsy in the vicinity of the base and new parameters. According to the Sobol theory, expected value of the sum of the total indices is 1.0.

Table xx. Total order Sobol indices for Georges Bank haddock. Estimates based on the Jar

Table entries represent fraction of total variance explained by eac

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	msy	1	0	0.00E+00	0	0.02816
base	msy	2	0.000161	1.77E-05	0	0.027685
base	msy	3	0.008906	1.91E-04	0	0.024137
base	msy	4	0.030937	2.07E-03	0	0.016217
base	msy	5	0.022955	2.15E-03	0	0.009055
base	msy	6	0.013268	1.54E-03	0	0.004727
base	msy	7	0.007464	1.04E-03	0	0.002397
base	msy	8	0.004083	7.07E-04	0	0.001153
base	msy	9	0.019813	4.48E-03	0	0.00448
base	Total	All	0.107585	0.012201	0	0.118011

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	ssb	0.07046	0.700836	0	0

Grand total: all parameters	1.009094

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	ssb	1	2.94E-07	0	2.94E-07	0.020336
base	ssb	2	3.29E-03	0.000198	3.28E-03	0.019765
base	ssb	3	1.88E-02	0.004328	1.88E-02	0.016064
base	ssb	4	1.74E-02	0.010035	1.74E-02	0.009796
base	ssb	5	1.10E-02	0.006684	1.10E-02	0.005445
base	ssb	6	6.40E-03	0.003526	6.38E-03	0.002889
base	ssb	7	3.60E-03	0.001803	3.60E-03	0.001486
base	ssb	8	1.97E-03	0.000885	1.98E-03	0.000729
base	ssb	9	9.74E-03	0.003583	9.90E-03	0.002986
base	Total	All	0.072179	0.031042	0.072274	0.079498

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	ssb	0.204616	0.506821	0.011275	0.014003

Grand total: all parameters	0.991708

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	msy	1	1.18E-06	6.98E-06	0	0.028145
new	msy	2	3.84E-05	2.49E-05	0	0.027861
new	msy	3	1.18E-03	7.71E-06	0	0.026518
new	msy	4	1.32E-02	7.37E-04	0	0.021624
new	msy	5	2.95E-02	2.18E-03	0	0.013994
new	msy	6	2.93E-02	1.95E-03	0	0.007022
new	msy	7	2.44E-02	1.96E-03	0	0.002606
new	msy	8	4.98E-03	6.00E-04	0	0.000859
new	msy	9	1.32E-02	1.99E-03	0	0.001991
new	total	All	0.115762	0.009457	0	0.130621

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
new	msy	0.050213	0.7015	0	0

Grand total: all parameters	1.007553

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	ssb	1	1.91E-06	1.04E-05	1.91E-06	0.021319
new	ssb	2	7.77E-04	9.23E-05	7.75E-04	0.020982
new	ssb	3	1.93E-02	7.24E-04	1.93E-02	0.018463
new	ssb	4	2.95E-02	3.96E-03	2.96E-02	0.01132
new	ssb	5	1.99E-02	6.04E-03	1.98E-02	0.005379
new	ssb	6	9.01E-03	5.04E-03	8.98E-03	0.002192
new	ssb	7	3.24E-03	4.09E-03	3.23E-03	0.000855
new	ssb	8	1.40E-03	8.48E-04	1.41E-03	0.000341
new	ssb	9	3.80E-03	2.03E-03	3.86E-03	0.000832
new	total	All	0.08693	0.022832	0.086907	0.081684

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	ssb	0.151187	0.531603	0.013724	0.014676

Grand total: all parameters	0.989542

Bluefish

Decomposition analyses was applied to data summarized in the 2017 and 2022 assessments of bluefish. Changes in average weight and selectivity vectors were relatively small. Selectivity estimates suggested a slightly greater dome in 2022 than in 2017. But the application of the Lorenzen method resulted in major changes in natural mortality rates (Table 5, Fig. 6) . The results of the Keyfitz model (Table 6) illustrate the overall drop in SBPR of 80%, from 4.14 to 0.65 kg/R and a 75% decline in YPR, from 0.66 kg/R to 0.15 kg/R. Changes in average weight **W** account for 10% of this change but the change in **M** alone reduces SBPR and YPR by 3.34 and 0.51 kg/R of their respective baseline values. The overall interaction term for SBPR ise much larger (=20%) than estimated for GB haddock. The interaction term for YPR is only about 1%.

TABLE 5. Bluefish input tables for Management Track Assessment in 2017 (baseline) and Research Track Assessment in 2022 (new).

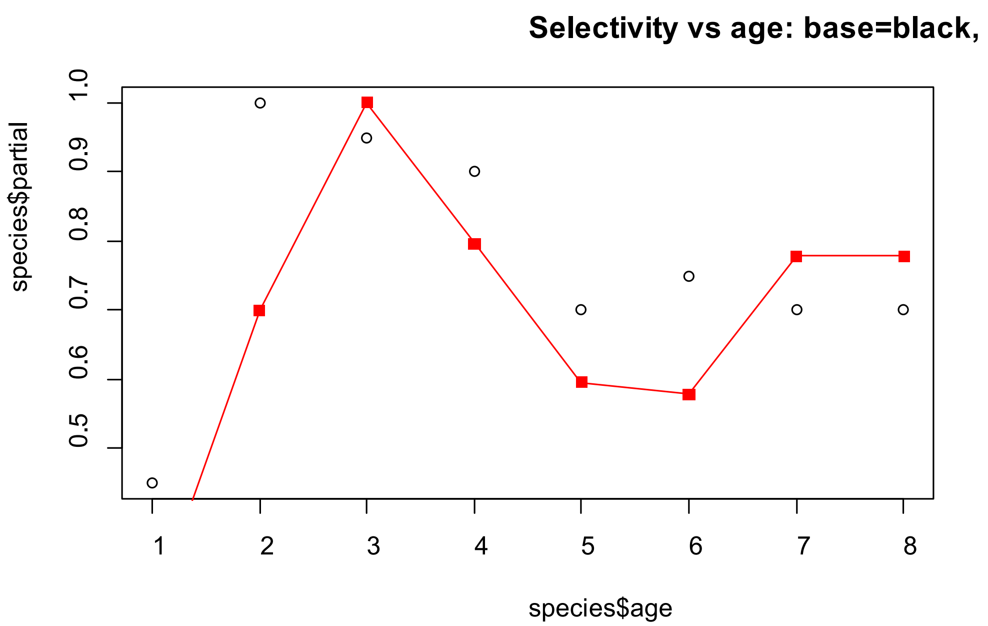
Table 2a. Summary of input data for Bluefish based o 2017 benchmark.

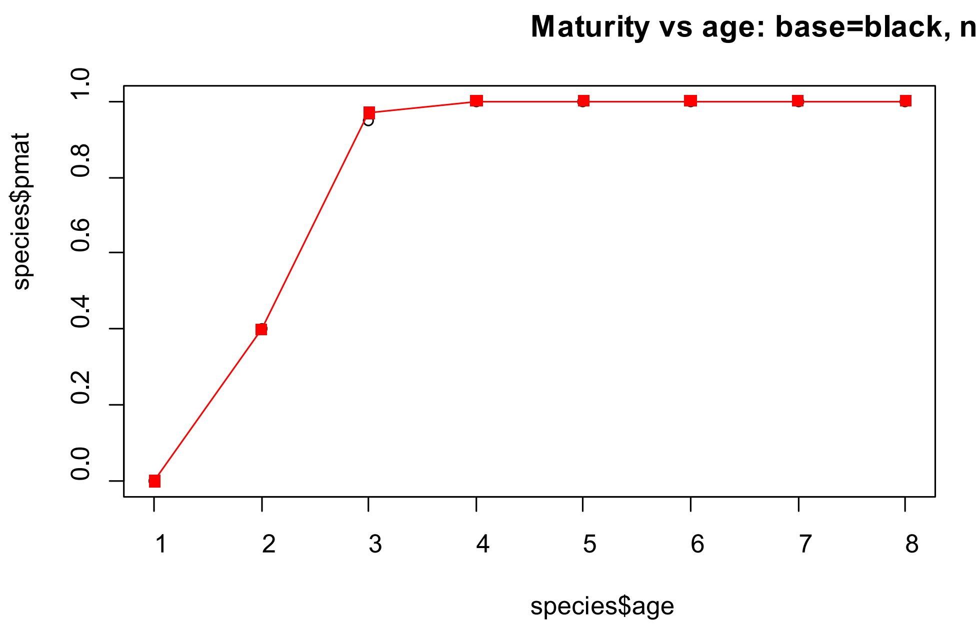
age	*ssbwgt*	*partial*	*pmat*	*M*
1	0.146087	0.45	0	0.2
2	0.438261	1	0.4	0.2
3	0.973913	0.95	0.95	0.2
4	1.947826	0.9	1	0.2
5	3.01913	0.7	1	0.2
6	4.041739	0.75	1	0.2
7	5.6	0.7	1	0.2
8	5.6	0.7	1	0.2

Table 2b. Summary of input data for Bluefish based on 2022 benchmark.

age	*ssbwgt*	*partial*	*pmat*	*M*
1	0.1374	0.262189	0	0.85
2	0.4052	0.700101	0.4	0.575
3	0.7724	1	0.97	0.453
4	1.39	0.796409	1	0.373
5	2.6772	0.595947	1	0.324
6	4.1084	0.578182	1	0.294
7	5.5444	0.777651	1	0.268
8	5.5444	0.777651	1	0.268


SSB wt vs age: base=black, new
species$ssbwgt
0
1
2
3
4
5
1
2
3
4
5
6
7
8
species$age


Selectivity vs age: base=black,
species$partial
0.5
0.6
0.7
0.8
0.9
1.0
1
2
3
4
5
6
7
8
species$age



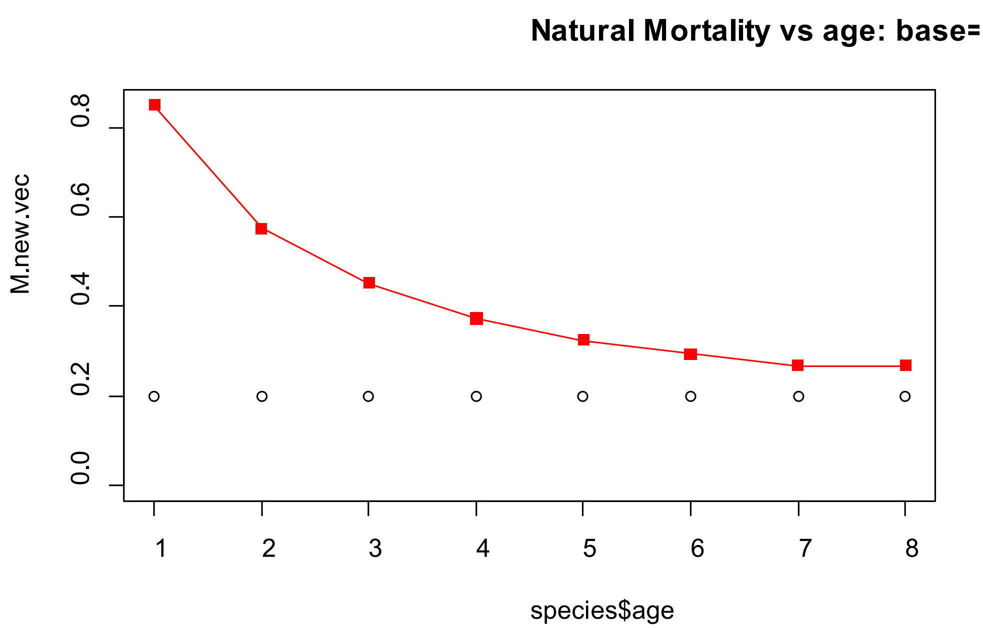
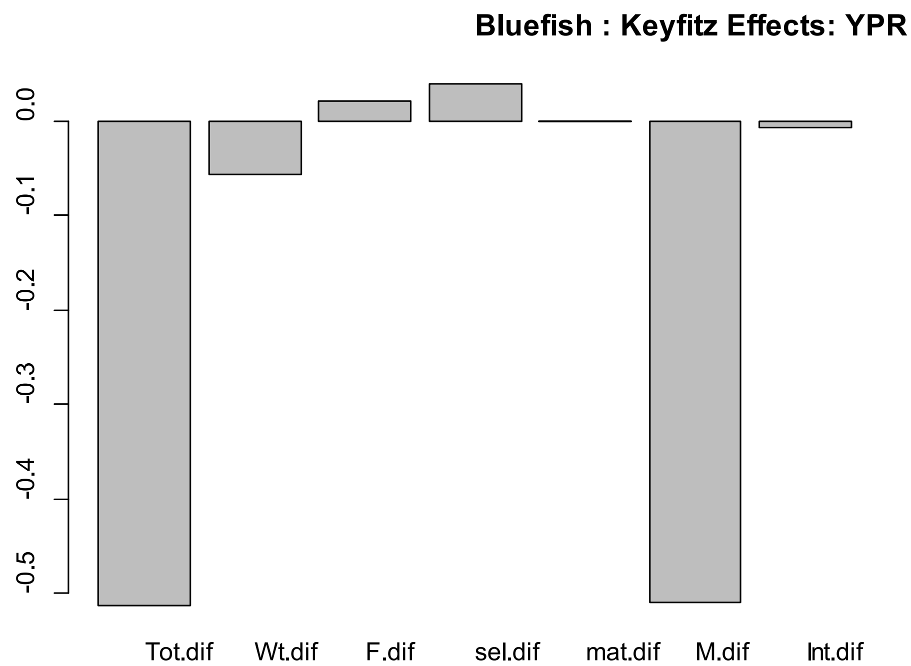


Figure 6. Comparison of changes in weight at age, selectivity at age, maturation at age, and natural mortality at age for bluefish between 2017 (black open dots) and 2022 (red closed squares). Input data for these plots are from Table 5.

Table 6. Decomposition of YPR and SBPR functions for Bluefish using the Keyfitz (left columns) and Horiuchi methods (right columns). The age-specific Horiuchi effects are presented in Table 7.

Keyfitz and Horiuchi Model Estimates of composite effects of Weight, Selectivity, maturation and M on SSB and BLUEFISH											
	KEYFITZ DELTA MODEL							***HORIUCHI MODEL***			
Factor	***Fmsp***	***SSBmsp***	***YPRmsp***					***SSBmsp***	***YPRmsp***		
Base Model	0.175536	4.144905	0.663611					4.144905	0.663611		
New Model	0.230297	0.649169	0.150945					0.649169	0.150945		
New - Base	0.054762	-3.49574	-0.51267	***Percent of total dif***				-3.49574	-0.51267	***Percent of total dif***	
First Order Effects				***SSBmsp***	***YPRmsp***					***SSBmsp***	***YPRmsp***
W		-0.30674	-0.056	9%	11%			-0.16078	-0.03365	5%	7%
F		-1.01691	0.021557	29%	-4%			-0.48917	0.020747	14%	-4%
Sel		0.44491	0.039552	-13%	-8%			0.213694	0.016141	-6%	-3%
F* Sel		-0.56056	0.075007	16%	-15%			-0.27547	0.036888	8%	-7%
Mat		0.007466	0	0%	0%			0.003888	0	0%	0%
M		-3.3403	-0.51079	96%	100%			-3.06336	-0.51591	88%	101%
Interaction 1		0.715844	-0.00698	-20%	1%			0	0	0%	0%
CHECK SUM		6.75E-07	9E-09					-3.3E-06	-2.9E-07		
2nd Order Effects		***SSBmsp***	***YPRmsp***	***Pred SSB***	***% dif***	***Pred YPR***	***% dif***				
W*F* Sel		-0.85038	0.009762	-0.87875	-3.3%	0.005106	47.7%				
W* Mat		-0.30082	-0.056	-0.29928	0.5%	-0.056	0.0%				
F* Sel *Mat		-0.55325	0.075007	-0.5531	0.0%	0.075007	0.0%				
W* M		-3.41568	-0.52654	-3.64704	-6.8%	-0.5668	-7.6%				
M* Mat		-3.33813	-0.51079	-3.33283	0.2%	-0.51079	0.0%				
F* Sel * M		-3.42561	-0.49434	-3.90086	-13.9%	-0.43579	11.8%				



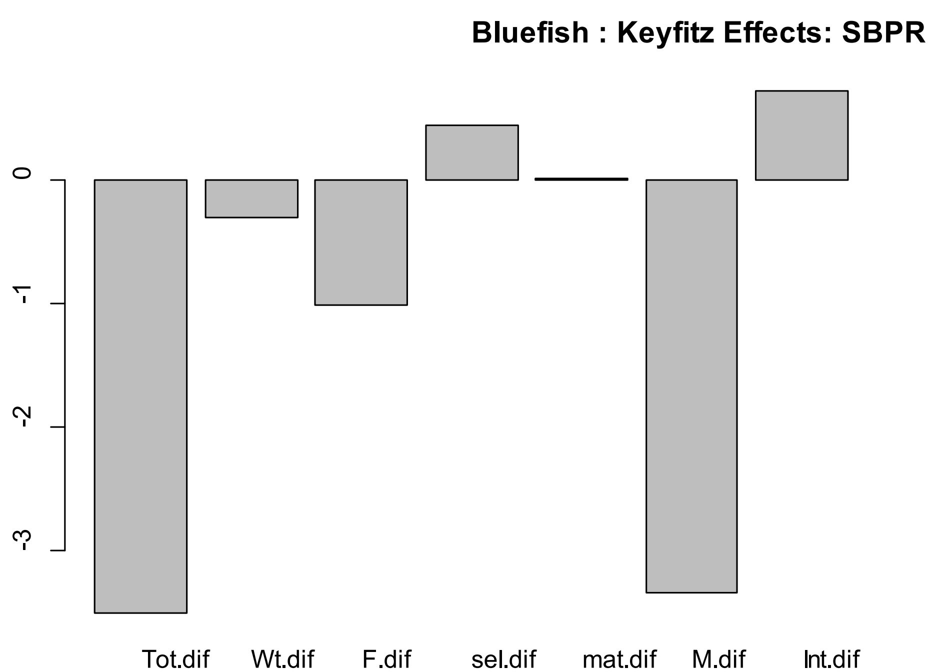


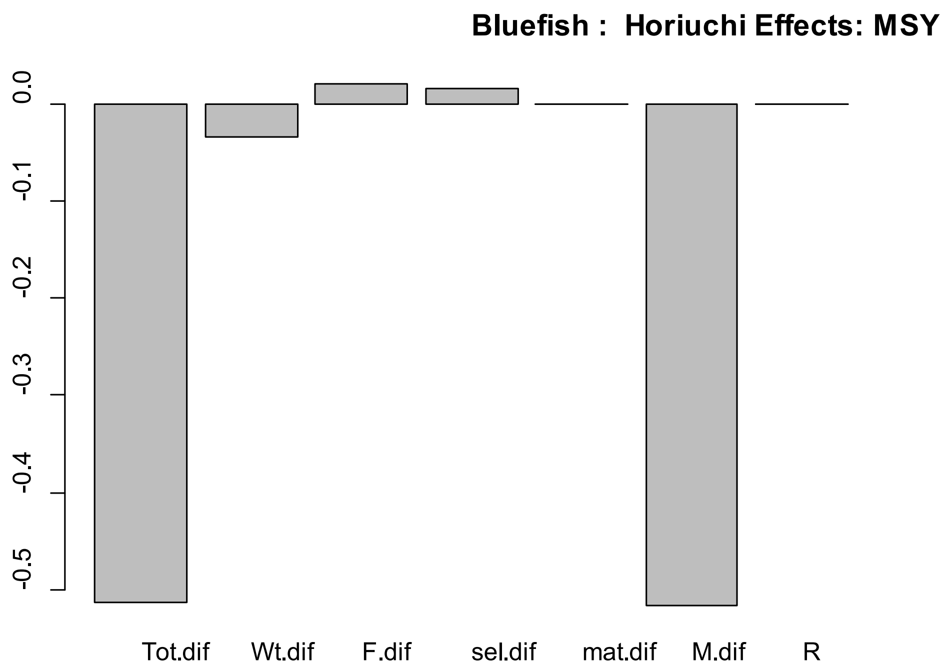
Figure 7. Graphical depiction of the Keyfitz decomposition of YPR (top) and SBPR(bottom) by parameter type for Bluefish. Units of y-axis are in kg. Estimates are summarized in Table 6.

Table 7. Decomposition of YPR (top) and SBPR (bottom) functions for Bluefish using the Horiuchi method by variable type and age. All table entries are in kg.

Table xx. Summary of Horiuchi effects for SSB and MSY by Weight, selectivity, maturity and natural Mortality by age for BLUEFISH.

Variable	***Age***	***Wt.ssb.H***	***Sel.ssb.H***	***pmat.ssb.***	***M.ssb.H***
SSB	1	0	0.068108	0	-1.20961
SSB	2	-0.00475	0.108139	0	-0.69397
SSB	3	-0.04199	-0.01722	0.003888	-0.44739
SSB	4	-0.07773	0.031695	0	-0.27209
SSB	5	-0.0324	0.026815	0	-0.16436
SSB	6	0.004367	0.035123	0	-0.09895
SSB	7	-0.00252	-0.01187	0	-0.05362
SSB	8	-0.00576	-0.0271	0	-0.12337
SSB	SUM	-0.16078	0.213694	0.003888	-3.06336
		Ffull	R.ave	pF	pM
SSB		-0.48917	0	0	0

Variable	***Age***	***Wt_Hor***	***Sel_Hor***	***pmat_Hor***	***M_Hor***
MSY	1	-0.00047	8.48E-03	0	-0.22115
MSY	2	-0.00247	7.83E-03	0	-0.11982
MSY	3	-0.01002	-4.83E-04	0	-0.07116
MSY	4	-0.01526	-9.07E-04	0	-0.04078
MSY	5	-0.00476	-2.43E-03	0	-0.02386
MSY	6	0.000663	-5.05E-03	0	-0.0143
MSY	7	-0.00041	2.66E-03	0	-0.00758
MSY	8	-0.00092	6.03E-03	0	-0.01726
MSY	SUM	-0.03365	0.016141	0	-0.51591
		Ffull	R.ave	pF	pM
MSY		0.020747	0	0	0



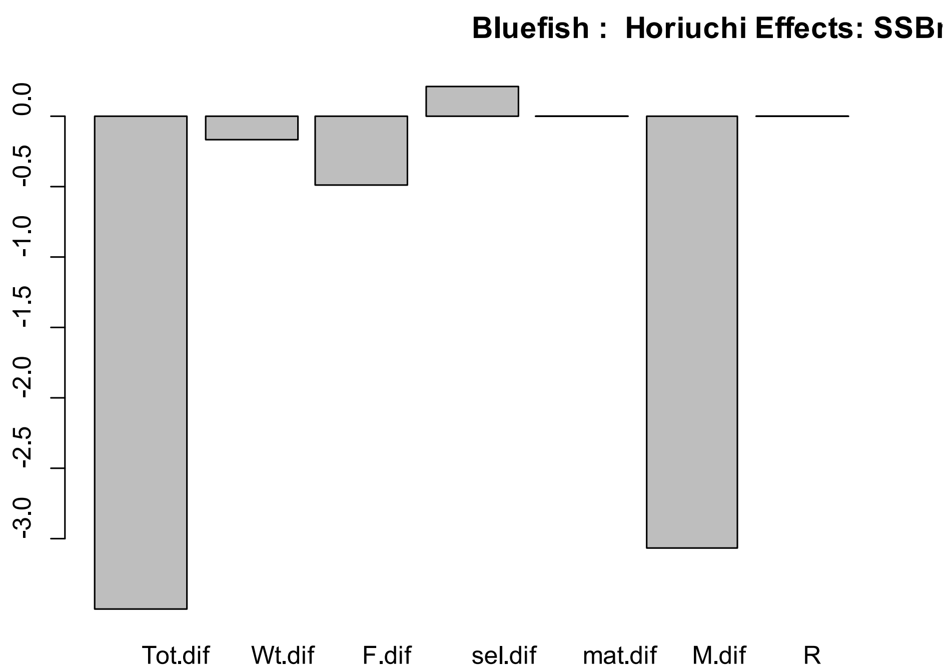


Figure 8. Graphical depiction of the Horiuchi decomposition of YPR (top) and SBPR(bottom) by parameter type for Bluefish . Units of y-axes are in kg. Estimates are summarized in Table 6 and 7.

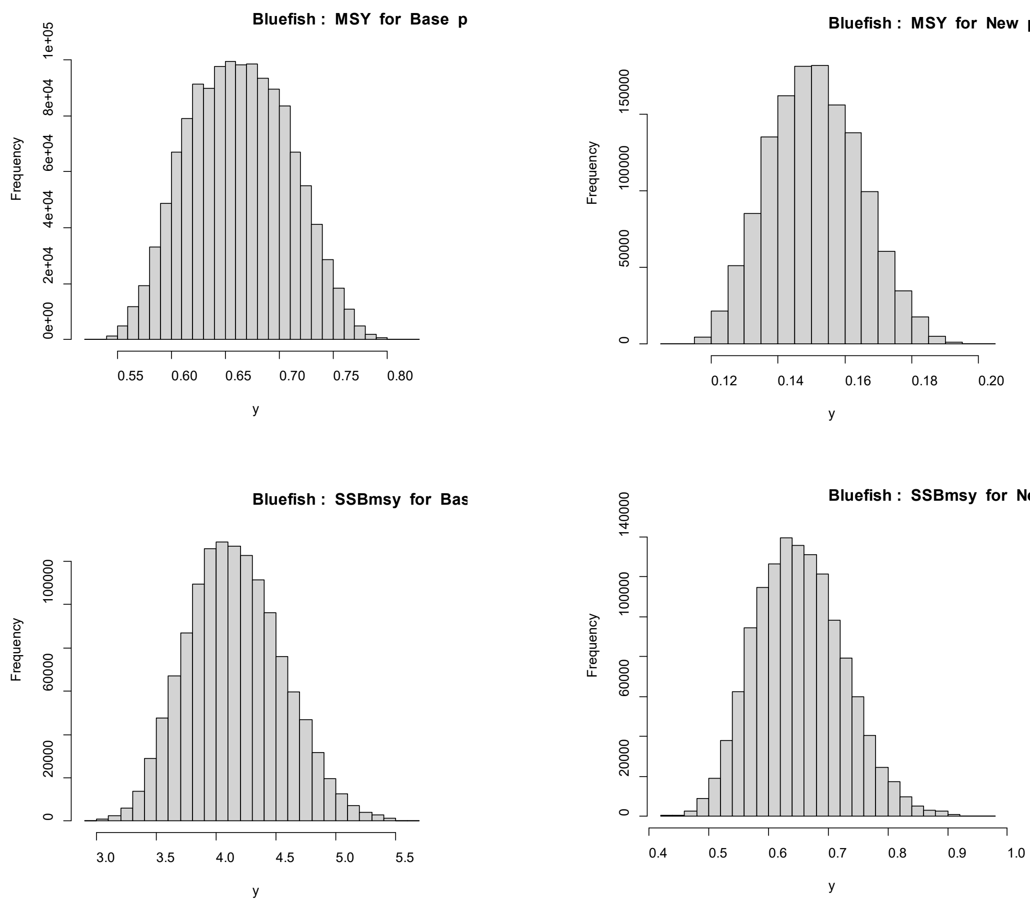


Figure 9. Distribution of MSY and SSBmsy for the baseline and new parameters for Bluefish based on the Sobol method. Variation of all input parameters by +/- 10%. Average recruitment is assumed to be 1.0 such that the above graphs are equivalent to the YPR and SBPR distributions, Units of x-axes are in kg.

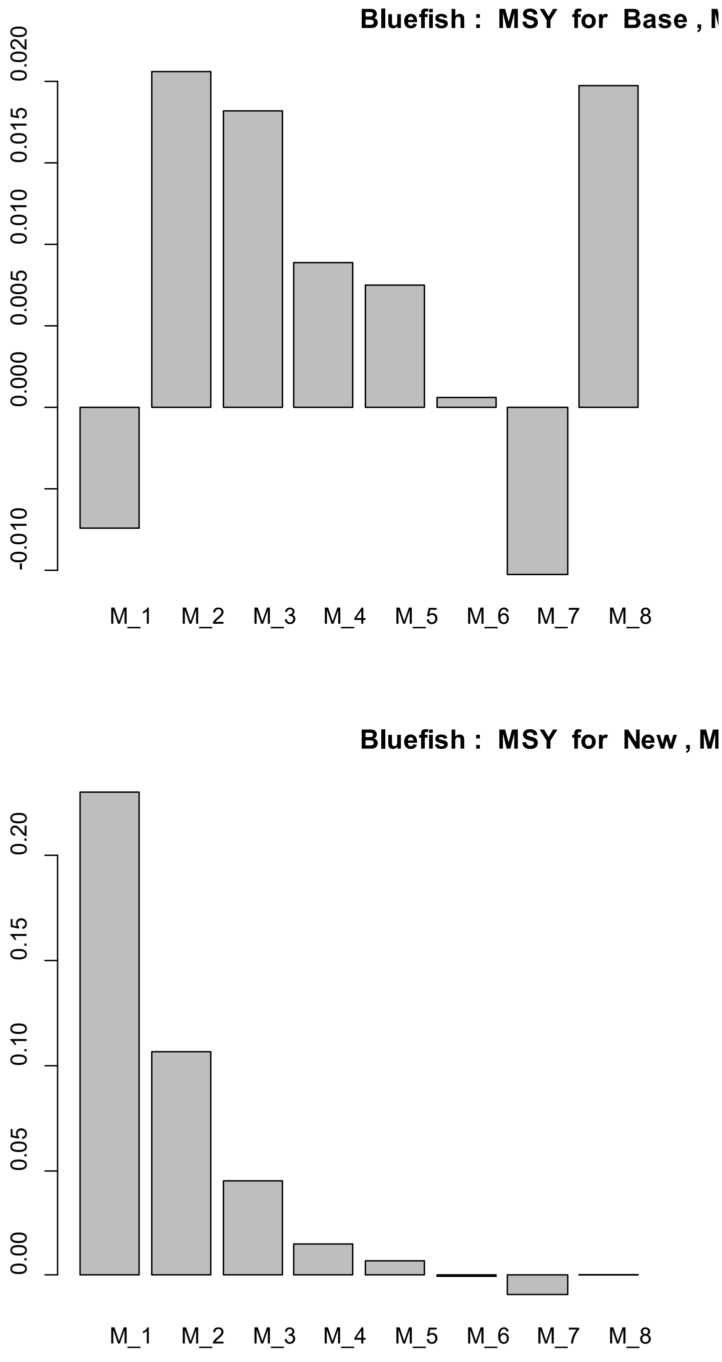


Figure 10. Comparison of the Sobol indices for Natural Mortality at age for the baseline (top) and new (bottom) parameter estimates for MSY for Georges Bank haddock. This figure demonstrates the local effect of system state on the Sobol indices; Sobol indices depict the sensitivity of the MSY model in the vicinity of the solution. Estimates are summarized in Table 8.

Table 8. Summary of age specific Sobol Total Order indices for Georges Bank. Table entries represent the fraction of total variance in MSY or SSBmsy explained by the input variables. Subtables illustrate the effects for MSY and SSBmsy in the vicinity of the base and new parameters. According to the Sobol theory, expected value of the sum of the total indices is 1.0.

Table xx. Total order Sobol indices for BLUEFISH. Estimates based on the Jansen algorithm.
Table entries represent fraction of total variance explained by each parameter.

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	msy	1	1.55E-04	2.69E-03	0	0.026324
base	msy	2	3.60E-03	5.85E-03	0	0.02402
base	msy	3	7.63E-03	9.29E-04	0	0.019601
base	msy	4	1.32E-02	4.71E-04	0	0.014338
base	msy	5	9.53E-03	1.47E-03	0	0.009731
base	msy	6	1.04E-02	2.54E-03	0	0.006176
base	msy	7	9.03E-03	3.50E-03	0	0.003475
base	msy	8	6.24E-02	2.42E-02	0	0.024047
base	Total	All	0.115978	0.041672	0	0.127711

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	msy	0.027793	0.670009	0	0

Grand total: all parameters	0.9832

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	ssb	1	0.00E+00	0.002332	0.00E+00	0.015001
base	ssb	2	2.05E-04	0.011384	2.06E-04	0.014976
base	ssb	3	2.75E-03	0.009608	2.73E-03	0.013947
base	ssb	4	5.93E-03	0.007059	5.94E-03	0.011431
base	ssb	5	7.26E-03	0.003109	7.39E-03	0.008318
base	ssb	6	6.88E-03	0.002339	6.93E-03	0.005511
base	ssb	7	6.92E-03	0.00124	6.90E-03	0.003317
base	ssb	8	4.82E-02	0.008736	4.81E-02	0.023317
base	Total	All	0.078118	0.045806	0.078142	0.095818

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	ssb	0.326085	0.378542	0.004568	0.010345

Grand total: all parameters	1.0174

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	msy	1	5.44E-04	2.31E-04	0	0.287507
new	msy	2	6.36E-03	2.26E-04	0	0.11061
new	msy	3	1.13E-02	8.48E-05	0	0.048092
new	msy	4	6.63E-03	1.19E-04	0	0.021477
new	msy	5	4.86E-03	7.31E-04	0	0.01064
new	msy	6	4.51E-03	1.14E-03	0	0.005401
new	msy	7	6.21E-03	2.25E-03	0	0.002239
new	msy	8	1.96E-02	7.14E-03	0	0.007091
new	total	All	0.060071	0.011928	0	0.493057

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
new	msy	0.028361	0.416845	0	0

Grand total: all parameters	1.0103

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	ssb	1	0	0.000984	0	0.194778
new	ssb	2	8.03E-04	0.006863	0.000804	0.088451
new	ssb	3	4.31E-03	0.012225	0.004281	0.047719
new	ssb	4	4.67E-03	0.005779	0.004678	0.024163
new	ssb	5	6.57E-03	0.002231	0.006686	0.012565
new	ssb	6	6.67E-03	0.001207	0.006715	0.006051
new	ssb	7	5.02E-03	0.001083	0.004998	0.002451
new	ssb	8	1.60E-02	0.00349	0.015946	0.007903
new	total	All	0.044044	0.033862	0.044108	0.384082

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
new	ssb	0.226437	0.27338	0.005419	0.020889

Grand total: all parameters	1.0322

Summer Flounder

The input data for Summer Flounder (Table 9) reveal changes in each of the **Θ** vectors except **M**. Weights at age **W** declined in the older ages in 2021 but increased slightly for younger fish. The partial recruitment vector **Sel** changed slightly with older fish having a slightly lower selectivity. Maturity at age **Mat** declined slightly for ages 1 and 2 in 2021 (Fig. 11).

The overall reduction in SBPR was about 0.388 kg/R or about a 26% reduction (Table 10). The corresponding change in YPR was only about 5%.

Most of the change in SBPR comes from the change in **F** (δ_F=-0.320 kg/R) with only minor effects from the change in weight (τ_W=-0.089 kg/R) or maturity (δ_{Mat}=-0.053 kg/R). The overall reduction on YPR is relatively minor (δ_T=-0.015 kg/R) due to the offsetting effects of reduction in W (δ_W=-0.046) and increases due to **F** (δ_F=0.009) and the large interaction term (δ_{Int}=0.018).

The Horiuchi estimates suggest that most of the reduction in MSY and YPR is due to the change in W but these changes are offset somewhat by the increased F and reduced selectivity on older fish.

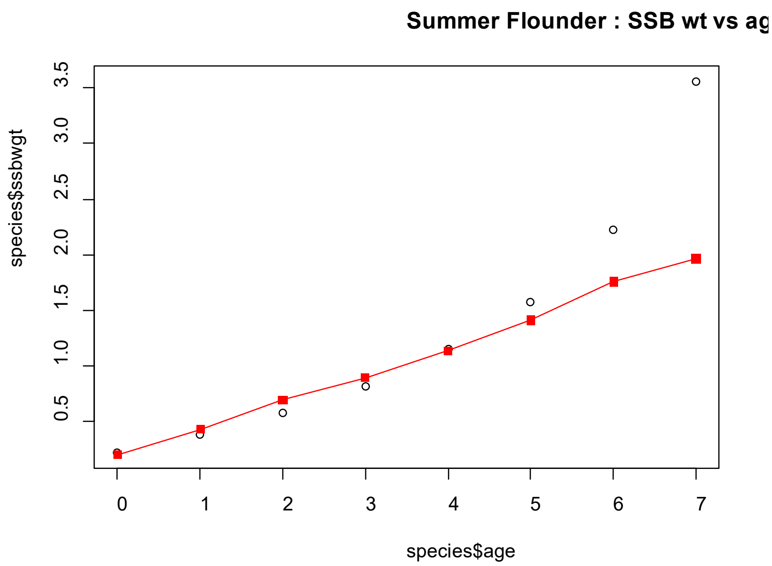
Table 9. Summer flounder INPUT TABLES for SARC Benchmark Assessment in 2013 (baseline) and Management Track Assessment in 2021 (new).

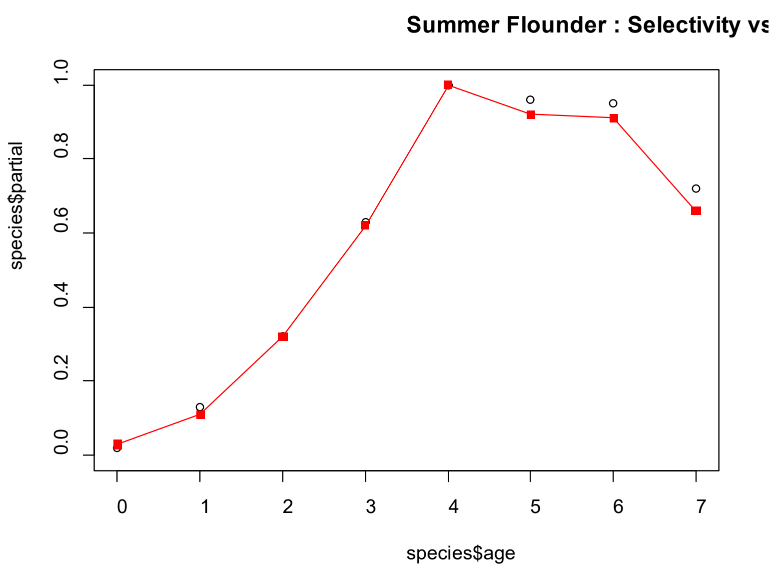
Table 3a. Summary of input data for Summer Flounder based on 2013 Benchmark

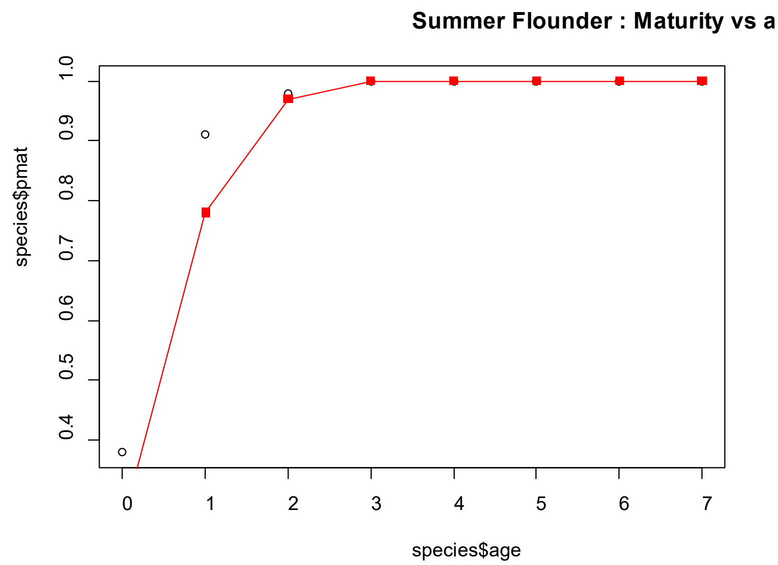
age	***ssbwgt***	***partial***	***pmat***	***M***
0	0.219	0.02	0.38	0.26
1	0.382	0.13	0.91	0.26
2	0.574	0.32	0.98	0.26
3	0.812	0.63	1	0.25
4	1.158	1	1	0.25
5	1.579	0.96	1	0.25
6	2.227	0.95	1	0.25
7	3.561	0.72	1	0.24

Table 3b. Summary of input data for Summer Flounder based on 2021 Update

age	***ssbwgt***	***partial***	***pmat***	***M***
0	0.201	0.03	0.26	0.26
1	0.431	0.11	0.78	0.26
2	0.693	0.32	0.97	0.26
3	0.895	0.62	1	0.25
4	1.137	1	1	0.25
5	1.413	0.92	1	0.25
6	1.758	0.91	1	0.25
7	1.964	0.66	1	0.24


Summer Flounder : SSB wt vs ag
species$ssbwgt
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
1
2
3
4
5
6
7
species$age


Summer Flounder : Selectivity vs
species$partial
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6
7
species$age



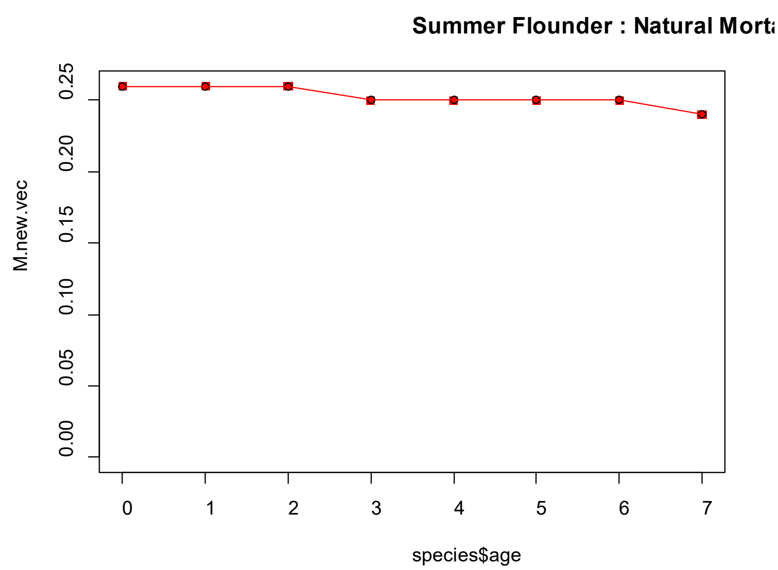


Figure 11. Comparison of changes in weight at age, selectivity at age, maturation at age, and natural mortality at age for Summer Flounder between 2013 (black open dots) and 2021 (red closed squares). Input data are listed in Table 9.

Table 10. Decomposition of YPR and SBPR functions for summer flounder using the Keyfitz (left columns) and Horiuchi methods (right columns). The age-specific Horiuchi effects are presented in Table 11.

Keyfitz and Horiuchi Model Estimates of composite effects of Weight, Selectivity, maturation and M on SSB and YPR.									SUMMER FLOUNDER			
	KEYFITZ DELTA MODEL								***HORIUCHI MODEL***			
Factor	***Fmsp***	***SSBmsp***	***YPRmsp***						***SSBmsp***	***YPRmsp***		
Base Model	0.314296	1.486005	0.338162						1.486005	0.338162		
New Model	0.448924	1.097881	0.322804						1.097881	0.322804		
New - Base	0.134628	-0.38812	-0.01536	***Percent of total dif***					-0.38812	-0.01536	***Percent of total dif***	
First Order Effects				***SSBmsp***	***YPRmsp***						***SSBmsp***	***YPRmsp***
W		-0.08939	-0.04063	23%	265%				-0.04749	-0.03252	12%	212%
F		-0.34568	0.011691	89%	-76%				-0.31198	0.020582	80%	-134%
Sel		0.035054	-0.00457	-9%	30%				0.025398	-0.00342	-7%	22%
F* Sel		-0.32029	0.009319	83%	-61%				-0.28658	0.017163	74%	-112%
Mat		-0.05312	0	14%	0%				-0.05405	0	14%	0%
M		0	0	0%	0%				0	0	0%	0%
Interaction 1		0.06501	0.018148	-17%	-118%				0	0	NA	NA
CHECK SUM		0	-1.2E-08						-8.3E-08	-6E-09		
2nd Order Effects		***SSBmsp***	***YPRmsp***	***Pred SSB***	***% dif***	***Pred YPR***	***% dif***					
W*F* Sel		-0.33316	-0.01536	-0.40001	-20.1%	-0.03351	-118.2%					
W* Mat		-0.14508	-0.04063	-0.14251	1.8%	-0.04063	0.0%					
F* Sel *Mat		-0.37274	0.009319	-0.37341	-0.2%	0.009319	0.0%					
W* M		-0.08939	-0.04063	-0.08939	0.0%	-0.04063	0.0%					
M* Mat		-0.05312	0	-0.05312	0.0%	0	0.0%					
F* Sel * M		-0.32029	0.009319	-0.32029	0.0%	0.009319	0.0%					

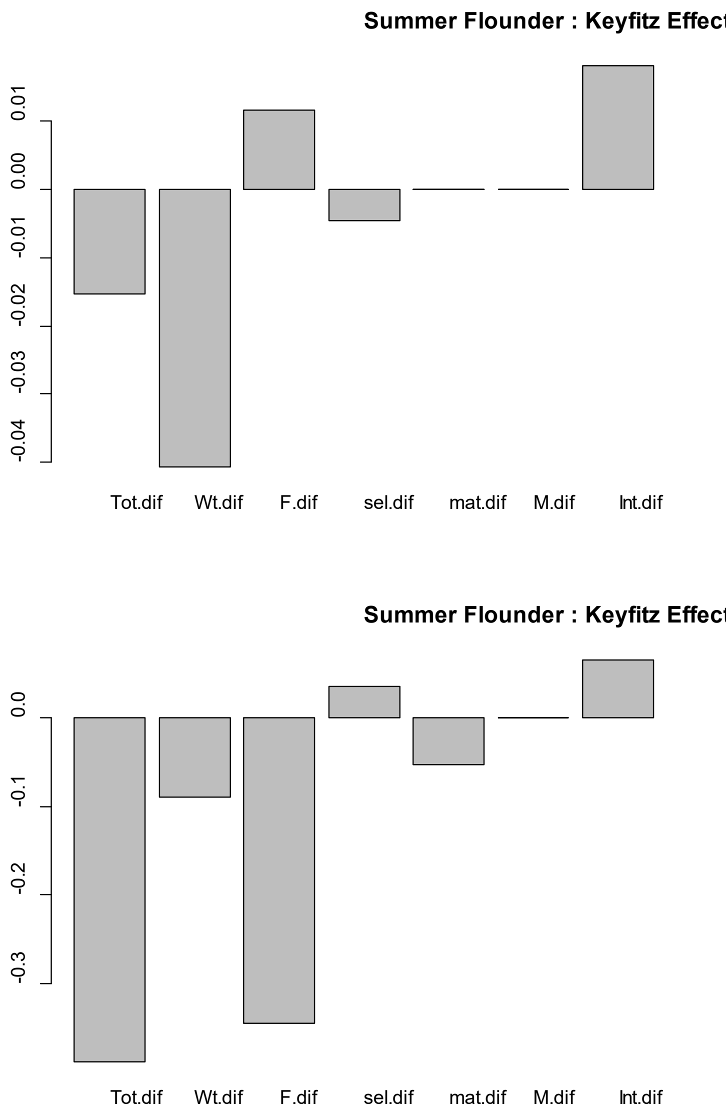


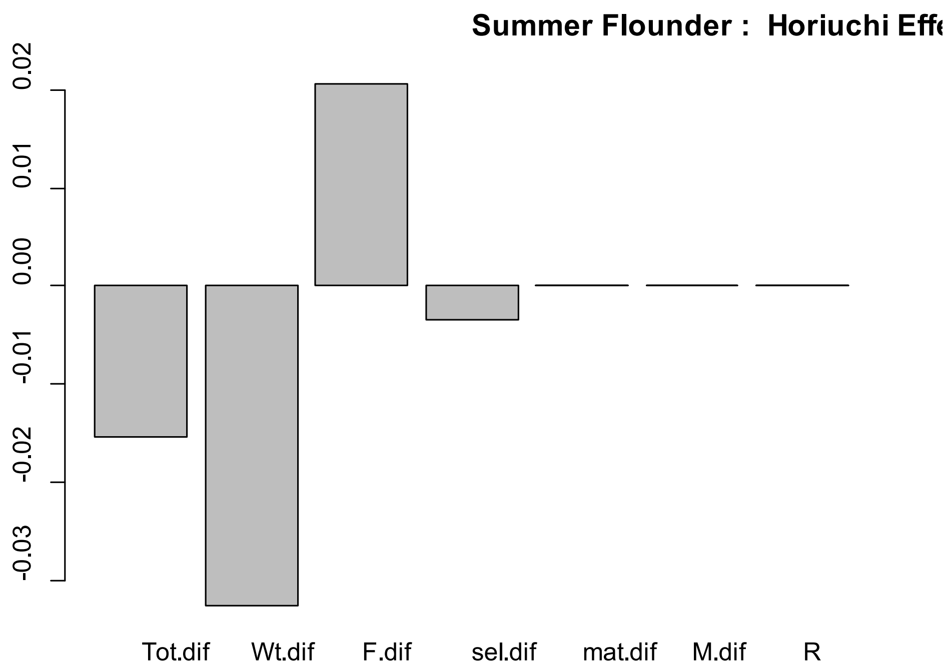
Figure 12. Graphical depiction of the Keyfitz decomposition of YPR (top) and SBPR(bottom) by parameter type for Summer Flounder. Units of y-axis are in kg. Estimates are given in Table 10.

Table 11. Decomposition of YPR (top) and SBPR (bottom) functions for Summer Flounder using the Horiuchi method by variable type and age. All table entries are in kg.

Table xx. Summary of Horiuchi effects for SSB and MSY by Weight, selectivity, maturity and natural Mortality by age for Summer Flounder.

Variable	***Age***	***Wt.ssb.H***	***Sel.ssb.H***	***pmat.ssb.***	***M.ssb.H***
SSB	1	-0.00461	-0.00476	-0.02015	0
SSB	2	0.02454	0.008917	-0.03131	0
SSB	3	0.047541	0	-0.00259	0
SSB	4	0.021259	0.002706	0	0
SSB	5	-0.00294	0	0	0
SSB	6	-0.01262	0.005335	0	0
SSB	7	-0.01955	0.003697	0	0
SSB	8	-0.10112	0.009502	0	0
SSB	SUM	-0.04749	0.025398	-0.05405	0
		Ffull	R.ave	pF	pM
SSB		-0.31198	0	0	0

Variable	***Age***	***Wt_Hor***	***Sel_Hor***	***pmat_Hor***	***M_Hor***
MSY	1	-0.00015	-5.51E-04	0	0
MSY	2	0.001469	3.99E-04	0	0
MSY	3	0.006788	0.00E+00	0	0
MSY	4	0.005996	-5.73E-05	0	0
MSY	5	-0.00139	0.00E+00	0	0
MSY	6	-0.00556	-4.28E-04	0	0
MSY	7	-0.00848	-4.90E-04	0	0
MSY	8	-0.03119	-2.29E-03	0	0
MSY	SUM	-0.03252	-0.00342	0	0
		Ffull	R.ave	pF	pM
MSY		0.020582	0	0	0



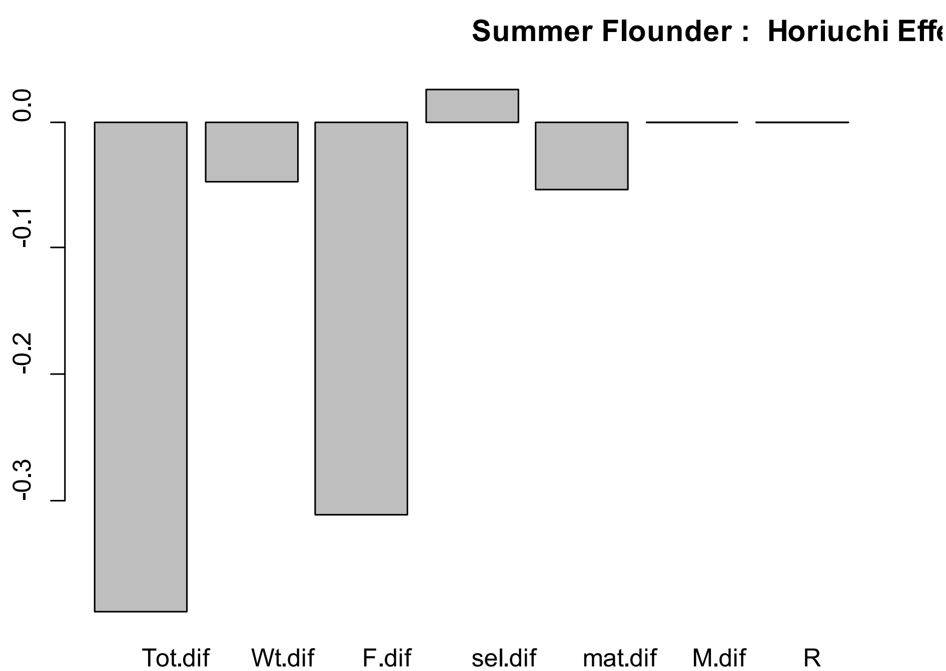


Figure 13. Graphical depiction of the Horiuchi decomposition of YPR (top) and SBPR(bottom) by parameter type for Summer Flounder . Units of y-axis are in kg. Estimates are given in Table 10 and 11.

Table 12. Summary of age specific Sobol Total Order indices for Georges Bank. Table entries represent the fraction of total variance in MSY or SSBmsy explained by the input variables. Subtables illustrate the effects for MSY and SSBmsy in the vicinity of the base and new parameters. According to the Sobol theory, expected value of the sum of the total indices is 1.0.

Table xx. Total order Sobol indices for Summer Flounder. Estimates based on the Jansen algorithm.

Table entries represent fraction of total variance explained by each param

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	msy	1	8.51E-06	4.93E-06	0	0.044459
base	msy	2	6.21E-04	6.37E-05	0	0.043059
base	msy	3	4.40E-03	1.09E-04	0	0.038079
base	msy	4	1.52E-02	1.03E-04	0	0.026704
base	msy	5	2.81E-02	2.23E-04	0	0.015925
base	msy	6	1.61E-02	4.02E-04	0	0.008056
base	msy	7	1.04E-02	8.79E-04	0	0.003787
base	msy	8	3.99E-02	1.07E-02	0	0.010653
base	Total	All	0.114738	0.012508	0	0.190722

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	msy	0.018869	0.661246	0	0

Grand total: all parameters	0.9981

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
base	ssb	1	0.000907	1.75E-05	0.000909	0.030005
base	ssb	2	0.008774	6.50E-04	0.00879	0.026585
base	ssb	3	0.011424	2.83E-03	0.011339	0.019067
base	ssb	4	0.010015	7.03E-03	0.010018	0.011344
base	ssb	5	0.006754	1.06E-02	0.006871	0.006773
base	ssb	6	0.00425	5.52E-03	0.004283	0.003894
base	ssb	7	0.002838	3.05E-03	0.002828	0.002172
base	ssb	8	0.020337	6.57E-03	0.020301	0.007538
base	Total	All	0.065299	0.036216	0.065339	0.107379

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
base	ssb	0.231324	0.454375	0.011156	0.019635

Grand total: all parameters	0.9907

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	msy	1	3.52E-05	2.44E-05	0	0.043462
new	msy	2	1.22E-03	2.05E-05	0	0.041345
new	msy	3	1.31E-02	2.18E-04	0	0.033504
new	msy	4	3.25E-02	7.94E-04	0	0.01913
new	msy	5	3.99E-02	1.77E-03	0	0.008068
new	msy	6	1.36E-02	1.05E-03	0	0.002641
new	msy	7	5.51E-03	8.28E-04	0	0.000795
new	msy	8	6.35E-03	1.30E-03	0	0.001289
new	total	All	0.112204	0.006004	0	0.150235

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
new	msy	0.028371	0.648754	0	0

Grand total: all parameters	0.9456

Dataset	Variable	Age	Ti.Wt	Ti.Sel	Ti.pmat	Ti.M
new	ssb	1	0.000654	8.12E-05	0.000655	0.030336
new	ssb	2	0.014747	9.59E-04	0.014773	0.026801
new	ssb	3	0.027222	5.14E-03	0.027018	0.016959
new	ssb	4	0.017523	8.95E-03	0.01753	0.007311
new	ssb	5	0.007316	9.39E-03	0.007443	0.002967
new	ssb	6	0.003048	3.10E-03	0.003071	0.001173
new	ssb	7	0.001278	1.18E-03	0.001272	0.000453
new	ssb	8	0.00311	1.36E-03	0.003104	0.00092
new	total	All	0.074897	0.030152	0.074866	0.08692

Dataset	Variable	Ti.Ffull	Ti.R.ave	Ti.pF	Ti.pM
new	ssb	0.185443	0.458781	0.01721	0.020205

Grand total: all parameters	0.9485

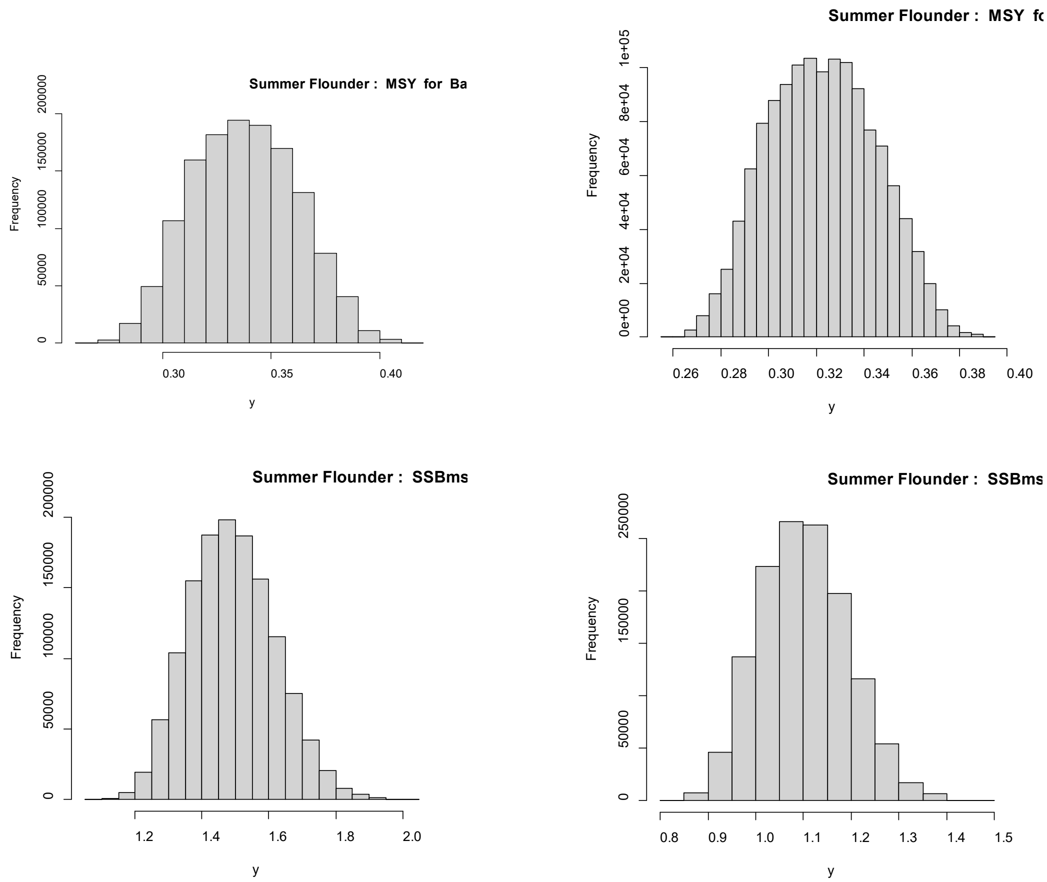


Figure 14. Distribution of MSY and SSBmsy for the baseline and new parameters for Summer Flounder based on the Sobol method. Variation of all input parameters by +/- 10%. Average recruitment is assumed to be 1.0 such that the above graphs are equivalent to the YPR and SBPR distributions, Units of x-axes are in kg.

Discussion

The three case studies suggest that single factor changes **M** for bluefish and **W** for Georges Bank haddock can dominate the overall effect of several factors and their interactions. The decomposition methods allow for identification of factors responsible for changes in productivity. In the case of bluefish, the justification for changing M was based on current theory of size-dependent predation as well as improved overall assessment model performance. Evidence of direct consumption by predators of bluefish may have also supported the change. For Georges Bank haddock the reduction in average weights at age has a profound effect on yield per recruit and SBPR. The observations reflect the effects of one or more factors such as competition for prey items and/or the effects of environment. Direct evidence for environmental effects could include thermal effects on bioenergetics or concentration of haddock into less productive habitats. In either case research efforts can be directed. Changes in selectivity and maturation have much less effect on the change in SBPR or YPR. While important, such changes have little effect on biological reference points or potential yield from the system.

The changes observed for Summer flounder illustrate the joint effect of shifts in fishing mortality and average weight. Reductions in average weight of age 6 and 7 fish result in a major decline in SBPR (τ_T=0.739) but very little change in YPR (τ_T=0.955). Interestingly, the interaction effect is greatest for summer flounder as is the relative prediction error associated with the joint effect of **F** and **W** for both SBPR and YPR (Table 12, 13).

Next Steps

This working paper is a work in progress. Additional details on the species examples will be provided. Literature citations will be added and cleaned up, as will the figures. The Horiuchi method in particular, provides useful, analytically sound, insights into the factors affecting overall measures of stock productivity (ie MSY and SSBmsy). Decomposition methods highlight the effects of changes in biological processes. Comparisons across multiple species will be helpful in identifying common patterns and possibly identifying common underlying ecological factors.

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APPENDIX 1. Equations for Keyfitz Difference and Multiplicative Decomposition Models.

Difference Model of Keyfitz

Overall Model for Spawning Biomass Per Recruit

δ_T = SBPR(W', F', Mat', M')-SBPR(W ,F, Mat, M)

First Order Effects

δ_W =SBPR(W', F, Mat, M) -SBPR(W ,F, Mat, M)
δ_F =SBPR(W, F', Mat, M) - SBPR(W ,F, Mat, M)
δ_{Mat} =SBPR(W, F, Mat', M) - SBPR(W ,F, Mat, M)
δ_M =SBPR(W, F, Mat, M') - SBPR(W ,F, Mat, M)

Second Order Effects

δ_{WF} =SBPR(W', F', Mat, M) - SBPR(W ,F, Mat, M)
δ_{WMat} =SBPR(W', F, Mat', M) - SBPR(W ,F, Mat, M)
δ_{WM} =SBPR(W', F, Mat, M') - SBPR(W ,F, Mat, M)
δ_{FMat} =SBPR(W, F', Mat', M) - SBPR(W ,F, Mat, M)
δ_{FM} =SBPR(W, F', Mat, M') - SBPR(W ,F, Mat, M)
δ_{MatM} =SBPR(W, F, Mat', M') - SBPR(W ,F, Mat, M)

Overall Model for Yield Per Recruit

δ_T = YPR(W', F', Mat', M')-YPR(W ,F, Mat, M)

First Order Difference Effects

δ_W =YPR(W', F, Mat, M) -YPR(W ,F, Mat, M)
δ_F =YPR(W, F', Mat, M) - YPR(W ,F, Mat, M)
δ_{Mat} =YPR(W, F, Mat', M) - YPR(W ,F, Mat, M)
δ_M =YPR(W, F, Mat, M') - YPR(W ,F, Mat, M)

Second Order Difference Effects

δ_{WF} =YPR(W', F', Mat, M) - YPR(W ,F, Mat, M)
δ_{WMat} =YPR(W', F, Mat', M) - YPR(W ,F, Mat, M)
δ_{WM} =YPR(W', F, Mat, M') - YPR(W ,F, Mat, M)
δ_{FMat} =YPR(W, F', Mat', M) - YPR(W ,F, Mat, M)
δ_{FM} =YPR(W, F', Mat, M') - YPR(W ,F, Mat, M)
δ_{MatM} =YPR(W, F, Mat', M') - YPR(W ,F, Mat, M)

Interactions for Difference Model

$$\delta_T = \delta_W + \delta_F + \delta_{Mat} + \delta_M + \xi_T$$

Two Way Interactions for Difference Model

$$\delta_{WF} = \delta_W + \delta_F + \xi_{WF}$$
$$\delta_{WMat} = \delta_W + \delta_{Mat} + \xi_{WMat}$$
$$\delta_{WM} = \delta_W + \delta_M + \xi_{WM}$$
$$\delta_{FMat} = \delta_F + \delta_{Mat} + \xi_{FMat}$$
$$\delta_{FM} = \delta_F + \delta_M + \xi_{FM}$$
$$\delta_{MatM} = \delta_{Mat} + \delta_M + \xi_{MatM}$$