Working Paper #17a Cusum for Seasonal Landings

In-Season Detection of System State

Ratio of annual with +/- 1 SD



Figure 1. Designation of good, average and poor fishing years based on total landings. The dashed red lines represent +/- 1 SD of the mean. Annual catches were normalized by dividing observations by the overall mean.

- Each state {Good, Average, Poor} has a characteristic asymptote and a slope.
 How much information is there in the slope?
- Can we make a decision before the season is over?



Comparison of Catch profiles for good, average and poor years: slopes at ~mid season



←YEAR: 1996-2019→

180	wit .	2006	80	199	599		00 <u>30</u>	1 .002	100	2004	205	300	100	300	209	200	269	200	30.1	2014	265	2.5	100	2.2	205
	1.5	1014	6.012	1.02	100	103	LOS	ENT.	LCO	100	110	EOE	1.00	111	660	LOU	110	600	1000	6.00	-	COL I	1.00	1000	1.00
2		LICE .	6.65	L CL	100	6.00	000	LEE .	LUC .	100	60%	6.00		600	000	LOL	610	600	LICE.	6.00	100	651	1.00	1000	100
1		1001	0.001	1.05	1.05	6.07	2,000	1.000	100	- 100		1001		00	000	LOCI		000	8000	L 05	LOO	600	1.00	100	, 100 ().
4	C	100	CDE	1.02	110	0.01	The			10	0.0	000	100	600	6601	100	ELSN .	600	1000	6.65		. 000	1.00	1311	X01
5	84 L	105		£ (52	100	1.00	Ine			00		000	1111		6601	100	66.00	680	1000	610	1.00	0.011	1.00	100	X DE
6		108	0.007	LOI	LCX	E CE	1000	1000		10	66.00	000		-	660	100	610	680	8000	610	1.00	600	1.00	100	
		LEE	LIN	C 11	105	1.01	chal	lenge	2.	an e	00				660		00	0.00	1000	6.00				- 1 	LW
		LUS	LON	LEN	LUL BUTT	1.00	1.00	0			1001	1.00			660	100		000		6.00		100		100	THE .
	2.00	100	0.00	6.000	1000	1.11	How	to d	etect		2023		1000			1.00	10.1	100				100			0.000
		E COL	CON .	6.00	100						101						1277			101	1.00	-			
		LOX	6.09		100	0.01	natt	ern e	arly		101	1.00	100		001	1.00	615.5	01.1		670	1.00	00	1.00	100	200
		100	CON.	1.00	LID	6.01	putt		Jurry	00	101	1.00	1000	1.00	001	1.00	630	1.01	100	600	1.00	000	100	100	200
- 34		000	600	1.00	100	0.01	in th		ar?	00	101	1.01	1000	1.00	0001	1.00	100	1.38	100	680	1.00	000	1.00	LOS	E OI
- 15	5	100	0.002	1.05	LID	6.00		ie ye	ai :	000	COL	0.30	1000	- 100		1.00	1991	1.00	100	680	1.00	000	0.05	100	200
	6	100	CO 1	0.07	8.092	1.00			10.0 M			1001			LL II	100		6.09	1000	6.00	100	001	1.00	1000	
- 17		1000	- 681	1.00	101	1.00	1000	600	600	100	660	001			LED	5 100	110	600	8000	6.00	100	001	0.36	1.87	1.00
	- <u>-</u>	101		6.00	LUSS	1.00			60	100	6320	604	ue	- 60	LIA	100		600	1000	610	- 100	001	1.00	1000	100
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				1.08	LOC				100	LON	ET DI				600		-	000		1.00		COLL .	1.00		1 1.30
	20	LUC .	-	10000	134	1.09				1.00	22.00	6.53		LUN	200	100	110	2.0.0			-	1000	0.000	1005	1.94
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		Constant Providence	100	1000	1414	1.30	1 100	-	1178	Contract of the	and the second s	109			110	100	1100	600	115.1	1.70	100	100	1.000	1.00	100
2		1504	111		2108	1.00	1.74	115	LCO	14	21.98	160	100	150	333	1.000		2.901	Inte	170	1.00	1.51	4.00	Contraction of the	Contraction of the
3		1.05	150	a state	3.938	3.33	000	670	118	1.00	110	533		600	400	1.00	1.0	100	6112	3.314	LIN	650	10	100	COLUMN TWO
2	1	2.381	100	1.04	1.704	1.05	2.98	101	62%	495	2031	100	100	3.054	100	1.568		2.06	1053	2,58	1.00	E.SM	2.65	1.1	E-ME 1
2	1	130	Link	505	190	2.60	0.00	1200	350	1.48	2470	334	6457	2.18	436	100	- 60 B	1576	101	2,234	1.00	105	6.65	2.00	10.00
3	9. B	LINE	2.03	1.000	1175	1.60	2.50	10%	103	1.00	2,150	3.00	<u>ш</u> ø	105	144	100	23150	1.08	EMB	170	1.00	130	1.00	1.00	8.00
3		1.502	- 696		148	2.35	100	1.11	LAN	100	20.8	3.300	1509	1.148	118	110	100	2540	3.652	158	1.34	2.40	1.00		
1		1.00		5 S.M.	156	2.06	1.00	LINE	1413	1.00	1011	1.00	00	2.58	15.0		3.488	2.04	LMB	Da	. 1.00	6.36			
E C		4.11	2.944		101	1.00	104		148		1394	LIN		1.76		1.04	1.11	1201	19.0	2.40	1.056	2.04	1.00		
		1000	1.01		183	1.00			100	Sec. 1	14.11	1000	13.7	100	DEPAYTOR .	1.00		100	1.21	2.00	LER	1007			
10		INC	1941	1.64	125	1.04		104	64.0	1000	10.00	1.003	1.4		4.55	6.90	6457	1496	THE .	1.00	1.00	1.000		605.7	
1		2.83	155	1.30	165	1.0%	000	LUNE .	LCO	1.44	2470	400	Contraction of the		404	LOW	110	1.00	1.31	101	100	6.001	Contraction of the	LINS	1.09
10	1	LIES	331	1.05	194	3.3.8	000	640	100	100	HX	3.04	2345	4.04	19.00	1.20	6402	4.50	677.6	1:00	100	3.81		105	0.057
		3.5%	6496	0.04	1.36	6.001	000	000	65.0	100	64.99	1.05	100	3.60	190	LIDI	1535	505	6.012	1.90	LO	100	LOI	100	1.00
38	<u>ار ا</u>	1.98	1.91	6,63	6.05	2.354	1000	600	110	1.995	2258	2.25	1404	240	44.5	100	2316	100	6.946	2.33	LINS	2.01	0.00	6003	1.01
	1	111	2454	6.02	LEN	0.00	1000	600	1444	100	6420	2.218	2011	1.01	2010	100	448	2.98	LOCE	6.30	1.00	00	0.00	1001	1.63
		E.MI	6.03	LO	, LLI	1.03	100		30.00	100	20.94	000	110	1.96	3.14	LO	6450	0.500	6.03	6.70	1.00	6.00	LOI	um	6.65
0		2.00	236	6.00	1.38	1.07			132	100	LE I			LN	LUX		00	130	E83		1.00	105	100	000	1 1.68
		1.00	con.	600	105	6,748	6.06	Line .	6436		0.0	1.51			138	1.001	ua	LOS		6.00		6.30	LOS	100	LOU
		1.1.1	6.53	1.00	100	6.000		109	N.M.	No. of Lot of Lo	10.3	0.00			100			100	1.06	LOC	0.02	LIN	LON	100	0.000
		1.80	6.02	6,000	1.00	6.000	n ive		1.341	1.17	CINI CINI	1.74	nel de l		COLOR.	100		600	LOC		0.00	6.000	0.000	LUD CTAR	6.00
	100	LSR	LOR	6.00	LID	1.05	100		0.019	1.00	101	LIN			100	LUD	-00-	000	1002	616	1.01	00	LOU	100	LIU
		101	111	1.00	Int	1.00			ELG.	LUU -		E M			100	LID	110	LOT .	LMT.	0.000	LU9	100	1.00	Int	1.03
	1.1	1016	1001	6.65	101	0.00	Lee .	00	LUCE	100		687			660	LO	40	600	100	6.00	LOC	101	0.00	109	0.00
1		100	0.035	6.03	101	1,00	100	1.51	ua	0.001	1.021	LOS	113			100		1.05	6.55	1.00	LO	001	1.00	115.0	- LIII 1/2
2		104	600	6,65	101	103	100	LOE	LCO	100		1001	111		105	6357		600	. IIII	6.001	103	1001	0.00	685	1,00
12	1	1000	6.00	LOX	101	0.01	001	00	100	100		000			660	100	110	600	Lini	6.00	100	100	1.00	1000	6.87
	1	8000	100	8,030	1000	101	0.000	600 C	0.00	100	0.0	1000		600	000	6.003	States in the second	000	BITC D	6.00	100	0.001	1.00	8000	1.11

←Week of the Year →

Potential Method for Detection: Cusum

- Method used in quality control for detection of change in underlying process.
- Change is expressed as a cumulative sum of deviations from the mean
- Easy to understand and visualize
- If everything is "normal" or in control, the sum should be around zero most of the time.

First let's consider a **constant mean and variance**. The upper and lower cusums are denoted as Cⁱ⁺ and Cⁱ⁻ and defined by the following recursive equations:

$$C_i^+ = \max \left[0, x_i - (\mu_0 + K) + C_{i-1}^+\right]$$
$$C_i^- = \max \left[0, (\mu_0 - K) - x_i + C_{i-1}^-\right]$$

- Starting values for C^{i+} and C^{i-} are both set to zero for i=1.
- The parameter **K** is a called the "slack" variable as is acts like a buffer or tolerance level. $\mathbf{K}=\delta\sigma$ represents the magnitude of the change one wishes to detect in μ .
- Changes in x^i of less than K are essentially zeroed out. For example $C^{i+1+} > C^{i+}$ increases only when $x^i > u^0 + K$ and $C^{i+1-} < C^{i-}$ only when $x^i < u^0 K$.
- Control bounds are set at a value **H** where $\mathbf{H} = \gamma \sigma$.
- Process is judged to be out of control when $C^{i+}>H$ or $C^{i-}<-H$.

Modification when mean and variance of landings L(w, y) varies with week (w):

$$C_{w,y}^{+} = \max\left[0, L_{w,y} - \left(\mu_{type,w} + K_{type,w}\right) + C_{w-1,y}^{+}\right]$$
$$C_{w,y}^{-} = \max\left[0, \left(\mu_{type,w} - K_{type,w}\right) - L_{w,y} + C_{i-1}^{-}\right]$$

Where $\mathbf{K}_{type,w} = \mathbf{k} \, \sigma_{type,w}$ where \mathbf{k} is a constant=1. \mathbf{H} is also redefined as $\mathbf{H}_{type,w} = \mathbf{h} \, \sigma_{type,w}$ where \mathbf{h} is a constant=5. Thus the Cusum process is specified to detect changes of 1 SD and is declared "out of control" when $\mathbf{C}_{w,y}^+ > \mathbf{H}_{type,w}$ or when $\mathbf{C}_{w,y}^- < -\mathbf{H}_{type,w}$. In other words, if the Cusum statistics lie outside the \mathbf{H} bounds, then one would reject the hypothesis that year in question was from a given type.

Characterizing the Seasonal pattern of landings for all years, good years, average years, and poor years.

Bounds represent +/- 1 SD of mean.

Now—compare each year to these 4 baselines using Cusum method.



Table 1. Summary of Cusum performance for detecting system state (good, average, poor) using slack variable K=1 SD and control bounds H=+/-5 SD limits. Entries represent the week when the Cusum first exceeded the control limit. The sign represents whether the Cusum statistics exceeded the upper bound (+) or fell below the lower bound (-).

		First Out of Bounds Detection Year						
	Classifica		Poor	Average	Good			
Year	tion	All Years	Years	Years	Years			
1996	Ave	43+	20+	44+	28-			
1997	Ave	none	27+	none	24-			
1998	Good	30+	20+	25+	none			
1999	Ave	none	30+	none	27-			
2000	Ave	none	36+	none	27-			
2001	Poor	none	none	none	24-			
2002	Poor	none	none	none	24-			
2003	Poor	none	43+	none	25-			
2004	Good	38+	21+	28+	39+			
2005	Ave	none	20+	none	28-			
2006	Ave	none	27+	none	28-			
2007	Ave	none	35+	none	24-			
2008	Ave	42+	31+	45+	24-			
2009	Ave	none	25+	none	28-			
2010	Ave	none	21+	none	none			
2011	Ave	none	20+	26+	39+			
2012	Ave	none	33+	none	27-			
2013	Poor	none	none	none	24-			
2014	Ave	none	33+	none	27-			
2015	Poor	none	none	none	24-			
2016	Poor	45+	none	none	24-			
2017	Good	38+	22+	32+	none			
2018	Good	29+	21+	27+	none			
2019	Good	31+	21+	27+	none			

Scoring a good year: 1998

		First O	ut of Bøun	ds Detectio	n Year	1
	Classifica		Poor	Average	Good	ĺ
Year	tion	All Years	Years	Years	Years	┢
1996	Ave	43+	20+	44+	28-	
1997	Ave	nøne	27+	none	24-	
1998	Good	30+	20+	25+	none	
1999	Ave	none	30+	none	27-	Γ
2000	Ave	none	36+	none	27-	
2001	Poor	none	none	none	24-	\mathbf{h}
2002	Poor	none	none	none	24-	
2003	Poor	none	43+	none	25-]
2004	Good	38+	21+	28+	39+	
2005	Ave	none	20+	none	28-	
2006	Ave	none	27+	none	28-	
2007	Ave	none	35+	none	24-	
2008	Ave	42+	31+	45+	24-	
2009	Ave	none	25+	none	28-]
2010	Ave	none	21+	none	none	
2011	Ave	none	20+	26+	39+]
2012	Ave	none	33+	none	27-	
2013	Poor	none	none	none	24-]
2014	Ave	none	33+	none	27-	
2015	Poor	none	none	none	24-	
2016	Poor	45+	none	none	24-]
2017	Good	38+	22+	32+	none]
2018	Good	29+	21+	27+	none]
2019	Good	31+	21+	27+	none	



Scoring a Poor Year 2016

		First Out of Bounds Detection Year						
	Classifica		Poor	Average	Good			
Year	tion	All Years	Years	Years	Years			
1996	Ave	43+	20+	44+	28-			
1997	Ave	none	27+	none	24-			
1998	Good	30+	20+	25+	none			
1999	Ave	none	30+	none	27-			
2000	Ave	none	36+	none	27-			
2001	Poor	none	none	none	24-			
2002	Poor	none	none	none	24-			
2003	Poor	none	43+	none	25-			
2004	Good	38+	21+	28+	39+			
2005	Ave	none	20+	none	28-			
2006	Ave	none	27+	none	28-			
2007	Ave	none	35+	none	24-			
2008	Ave	42+	31+	454	24-			
2009	Ave	none	25+	none	28-			
2010	Ave	none	21+	none	none			
2011	Ave	none	20+	26+	39+			
2012	Ave	none	33+	none	27-			
2013	Poor	none	none	none	24-			
2014	Ave	none	33+	none	27-			
2015	Poor	none	none	none	24-			
2016	Poor	45+	none	none	24-			
2017	Good	38+	22+	32+	none			
2018	Good	29+	21+	27+	none			
2019	Good	31+	21+	27+	none			



Scoring the best year: 2019

		First Out of Bounds Detection Year						
	Classifica		Poor	Average	Good]		
Year	tion	All Years	Years	Years	Years			
1996	Ave	43+	20+	44+	28-]		
1997	Ave	none	27+	none	24-	Y		
1998	Good	30+	20+	25+	none]		
1999	Ave	none	30+	none	27/]		
2000	Ave	none	36+	none	27-]		
2001	Poor	none	none	none	24-]		
2002	Poor	none	none	none	24-			
2003	Poor	none	43+	none	25-]		
2004	Good	38+	21+	28+	39+]		
2005	Ave	none	20+	none	28-]		
2006	Ave	none	27+	none	28-	Y		
2007	Ave	none	35+	none	24-]		
2008	Ave	42+	31+	45+	24]		
2009	Ave	none	25+	none	2/8-]		
2010	Ave	none	21+	none	none]		
2011	Ave	none	20-	26+	39+]		
2012	Ave	none	33+	none	27-]		
2013	Poor	none	none	none	24-			
2014	Ave	none	33+	none	27-	ľ		
2015	Poor	none	none	none	24-	Į		
2016	Poor	45+	none	none	24-]		
2017	Good	38+	22+	32+	none	Į		
2018	Good	29+	21+	27+	none	Į		
2019	Good	31+	21+	27+	none	J		



Scoring an average year: 2014

		First Out of Bounds Detection Year						
	Classifica		Poor	Average	Good			
Year	tion	All Years	Years	Years	Years			
1996	Ave	43+	20+	44+	28-			
1997	Ave	none	27+	none	24-			
1998	Good	30+	20+	25+	none			
1999	Ave	none	30+	none	27-			
2000	Ave	none	36+	none	27-			
2001	Poor	none	none	none	24-			
2002	Poor	none	none	none	24-			
2003	Poor	none	43+	none	25-			
2004	Good	38+	21+	28+	39+			
2005	Ave	none	20+	none	28-			
2006	Ave	none	27+	none	28-			
2007	Ave	none	35+	none	24-			
2008	Ave	42+	31+	45+	24-			
2009	Ave	none	25+	none	28-			
2010	Ave	none	21+	none	none			
2011	Ave	none	20+	26+	39+			
2012	Ave	none	33+	none	27-			
2013	Poor	none	none	none	24-			
2014	Ave	none	33+	none	27-			
2015	Poor	none	none	none	24-			
2016	Poor	45+	none	none	24-			
2017	Good	38+	22+	32+	none			
2018	Good	29+	21+	27+	none			
2019	Good	31+	21+	27+	none			



Sources of Uncertainty

- Basis for original classification of "Good", "Average", "Poor"
- Specification of seasonal means and variances based on samples
- Setting the control limits for a time varying mean and variance.