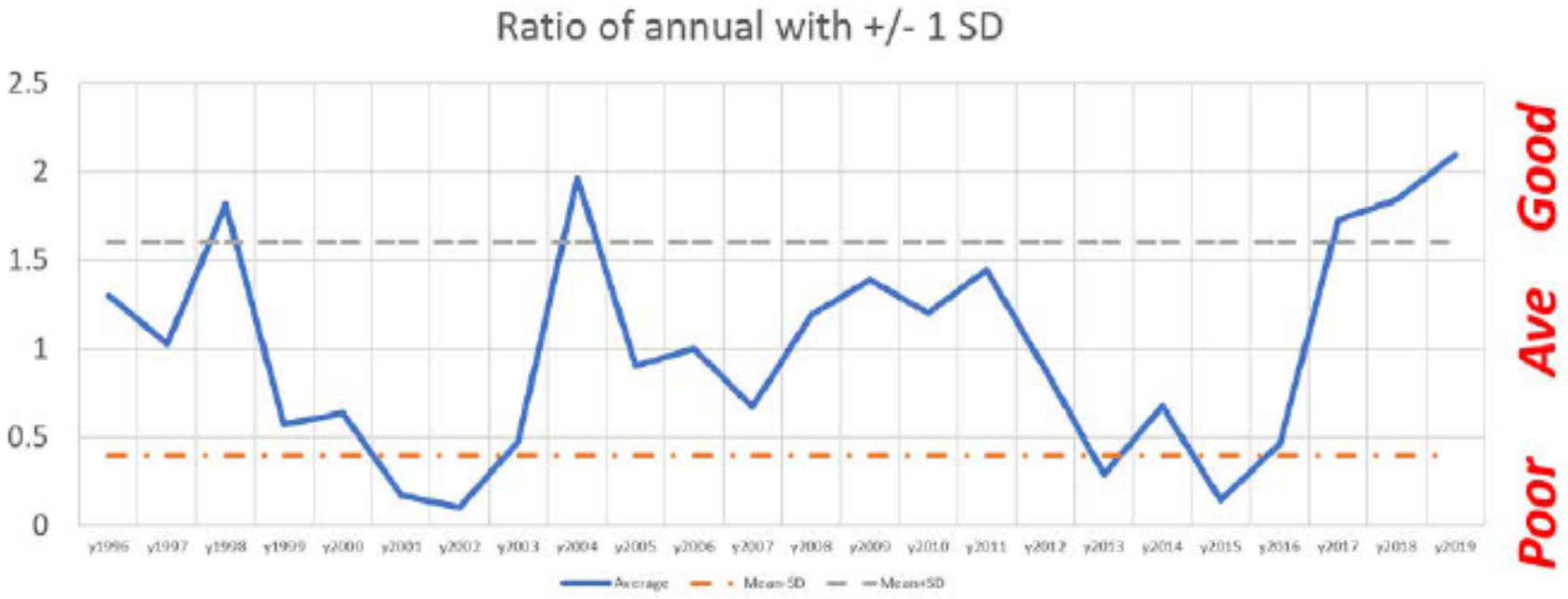
Working Paper #17a
Cusum for Seasonal Landings

In-Season Detection of System State



Good{1998, 2004, 2017, 2018, 2019}

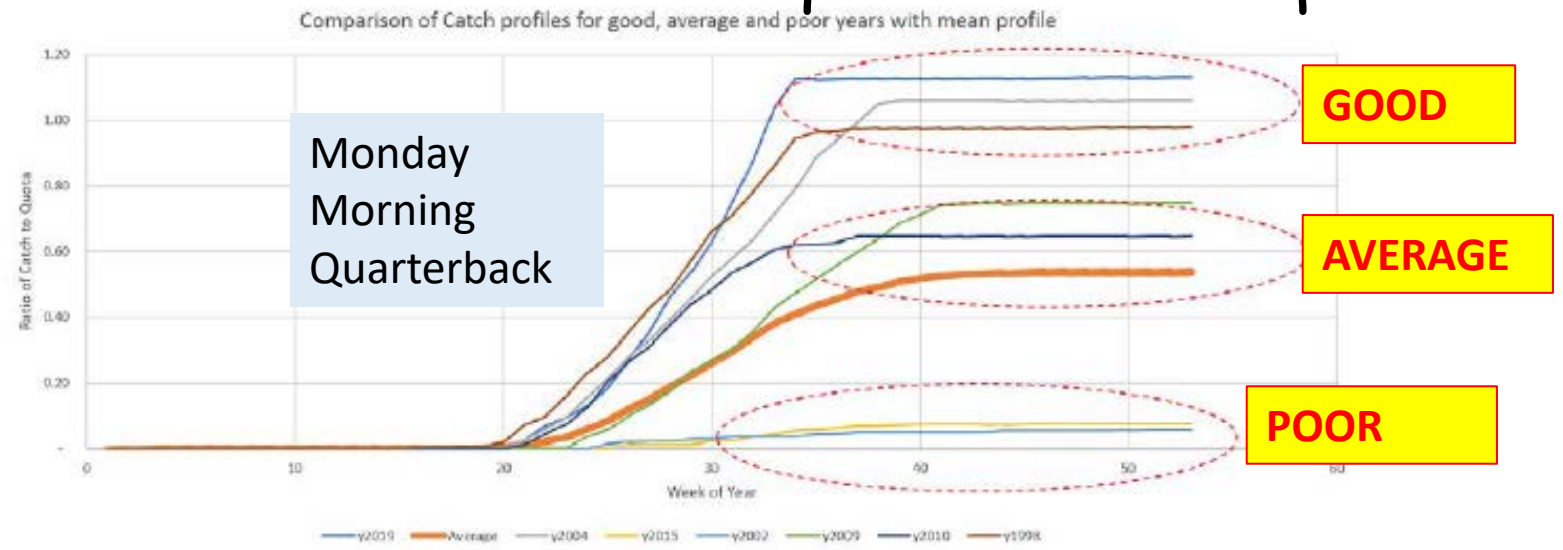
Average {1997, 1999, 2000, 2005-2007, 2010-2012, 2014}

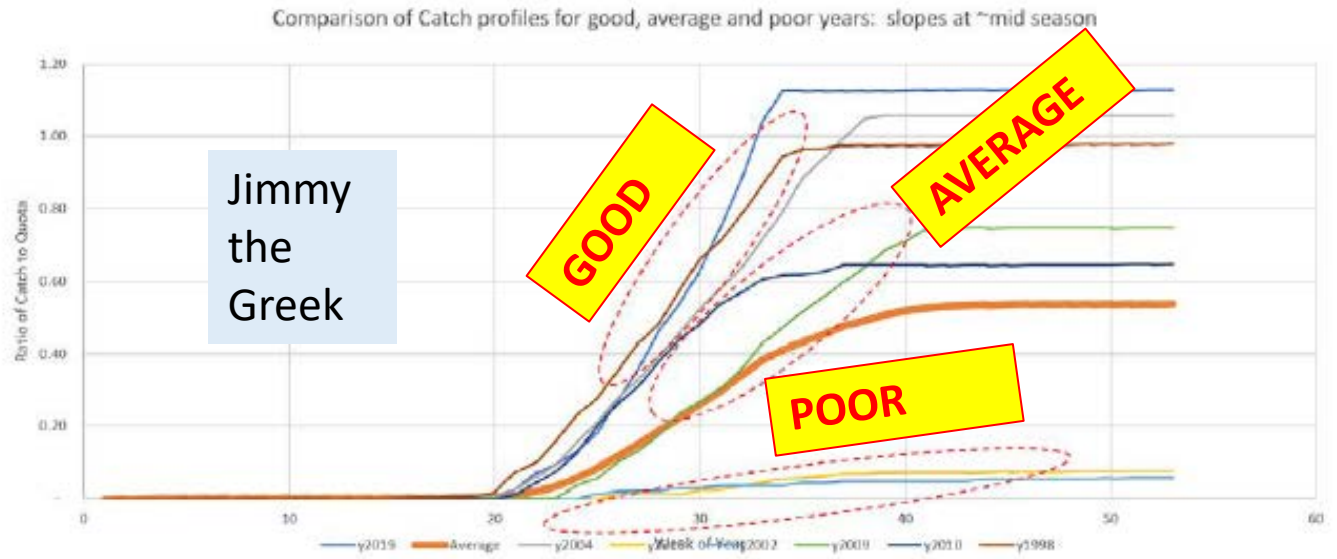
Poor {2001,2002, 2003, 2013, 2015, 2016}

Figure 1. Designation of good, average and poor fishing years based on total landings. The dashed red lines represent +/- 1 SD of the mean. Annual catches were normalized by dividing observations by the overall mean.

- Each state {Good, Average, Poor} has a characteristic asymptote and a slope.
- How much information is there in the slope?
- Can we make a decision before the season is over?

late

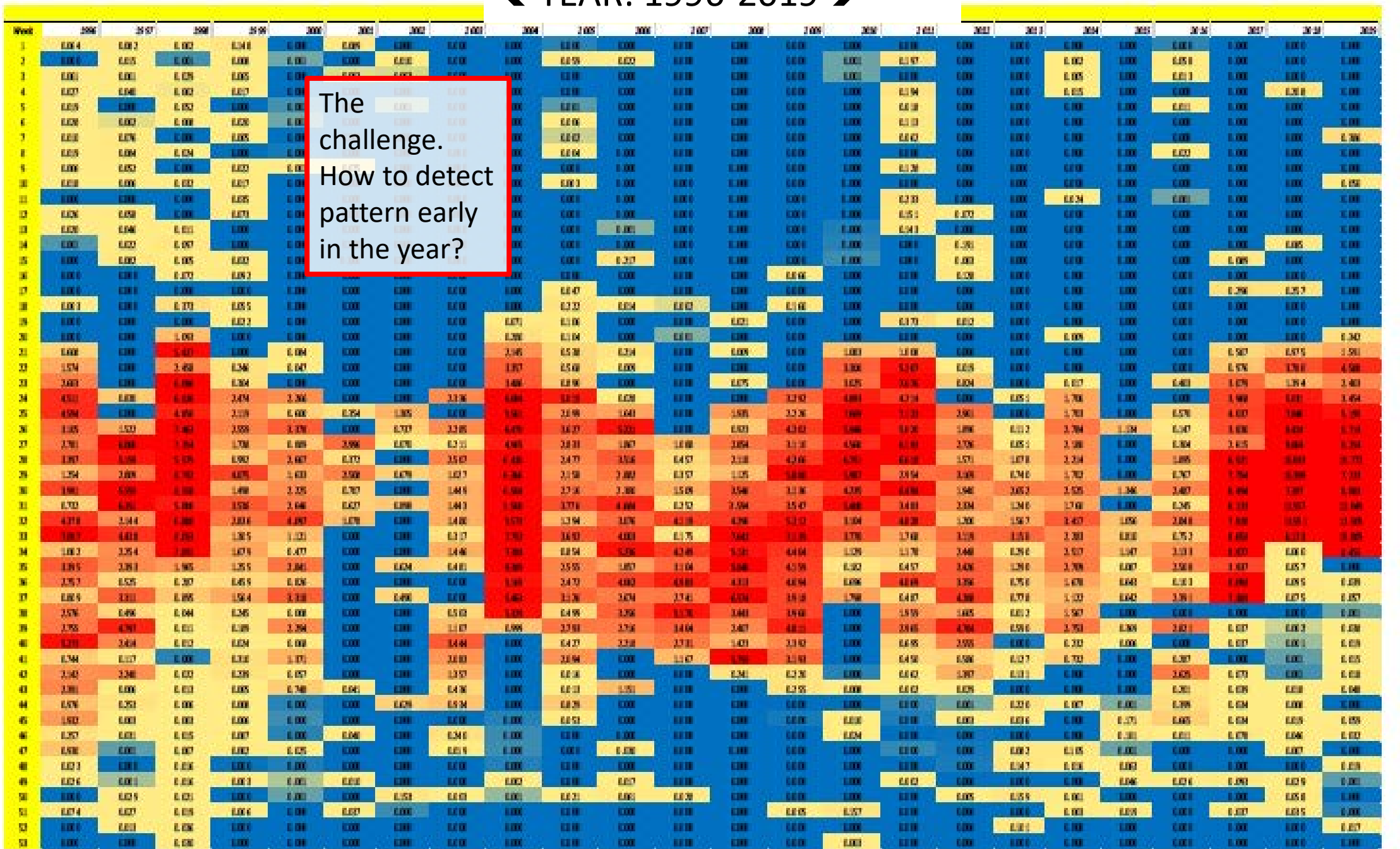




early

←YEAR: 1996-2019→

←Week of the Year→



Potential Method for Detection: Cusum

- Method used in quality control for detection of change in underlying process.
- Change is expressed as a cumulative sum of deviations from the mean
- Easy to understand and visualize
- If everything is “normal” or in control, the sum should be around zero most of the time.

First let's consider a **constant mean and variance**. The upper and lower cusums are denoted as $\mathbf{C^{i+}}$ and $\mathbf{C^{i-}}$ and defined by the following recursive equations:

$$C_i^+ = \max\left[0, x_i - (\mu_0 + K) + C_{i-1)}^+\right]$$

$$C_i^- = \max\left[0, (\mu_0 - K) - x_i + C_{i-1)}^-\right]$$

- Starting values for $\mathbf{C^{i+}}$ and $\mathbf{C^{i-}}$ are both set to zero for i=1.
- The parameter **K** is a called the "slack" variable as is acts like a buffer or tolerance level. $\mathbf{K}=\delta\sigma$ represents the magnitude of the change one wishes to detect in μ.
- Changes in $\mathbf{x^i}$ of less than **K** are essentially zeroed out. For example $\mathbf{C^{i+1+}} > \mathbf{C^{i+}}$ increases only when $\mathbf{x^i > u^0 + K}$ and $\mathbf{C^{i+1-}} < \mathbf{C^{i-}}$ only when $\mathbf{x^i < u^0 - K}$.
- Control bounds are set at a value **H** where $\mathbf{H}=\gamma\sigma$.
- Process is judged to be out of control when $\mathbf{C^{i+} > H}$ or $\mathbf{C^{i-} < -H}$.

Modification when mean and variance of landings *L(w,y)* varies with week (*w*):

$$C^{+}_{w,y} = \max\left[0, L_{w,y} - \left(\mu_{type,w} + K_{type,w}\right) + C^{+}_{w-1,y)}\right]$$

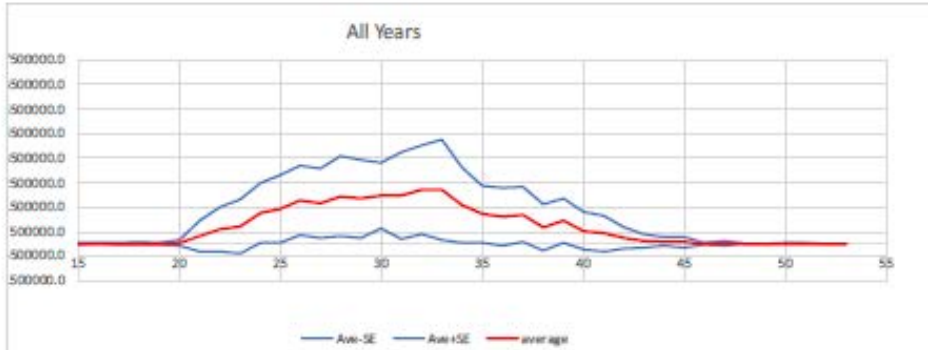
$$C^{-}_{w,y} = \max\left[0, \left(\mu_{type,w} - K_{type,w}\right) - L_{w,y} + C^{-}_{i-1)}\right]$$

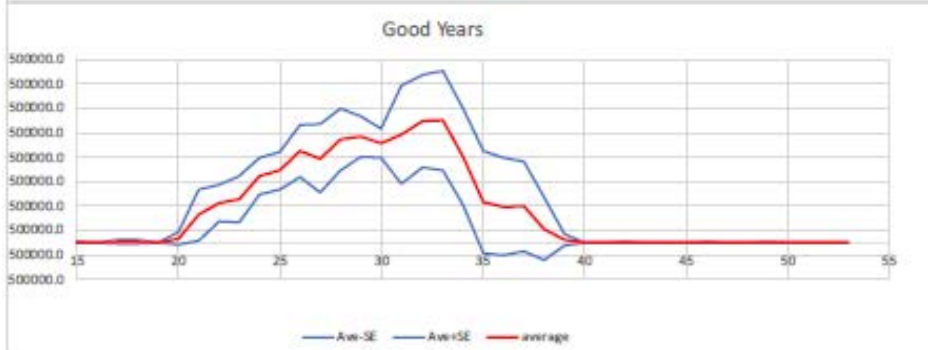
Where $\mathbf{K_{type,w}} = \mathbf{k}\,\boldsymbol{\sigma}_{\mathbf{type,w}}$ where **k** is a constant=1. **H** is also redefined as $\mathbf{H_{type,w}} = \mathbf{h}\,\boldsymbol{\sigma}_{\mathbf{type,w}}$ where **h** is a constant=5. Thus the Cusum process is specified to detect changes of 1 SD and is declared "out of control" when $\mathbf{C_{w,y}}^{+} > \mathbf{H_{type,w}}$ or when $\mathbf{C_{w,y}}^{-} < -\mathbf{H_{type,w}}$. In other words, if the Cusum statistics lie outside the **H** bounds, then one would reject the hypothesis that year in question was from a given type.

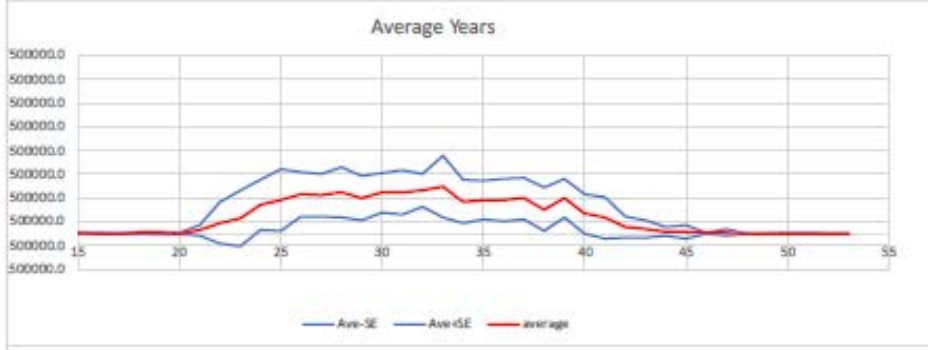
Characterizing the Seasonal pattern of landings for all years, good years, average years, and poor years.

Bounds represent +/- 1 SD of mean.

Now—compare each year to these 4 baselines using Cusum method.







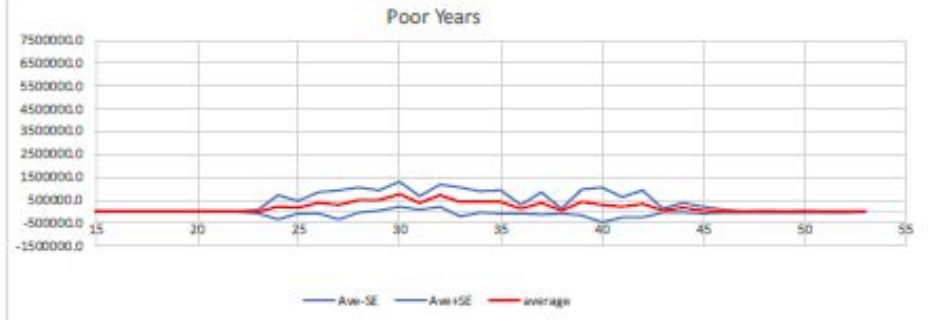


Table 1. Summary of Cusum performance for detecting system state (good, average, poor) using slack variable K=1 SD and control bounds H=+/-5 SD limits. Entries represent the week when the Cusum first exceeded the control limit. The sign represents whether the Cusum statistics exceeded the upper bound (+) or fell below the lower bound (-).

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

Scoring a good year: 1998

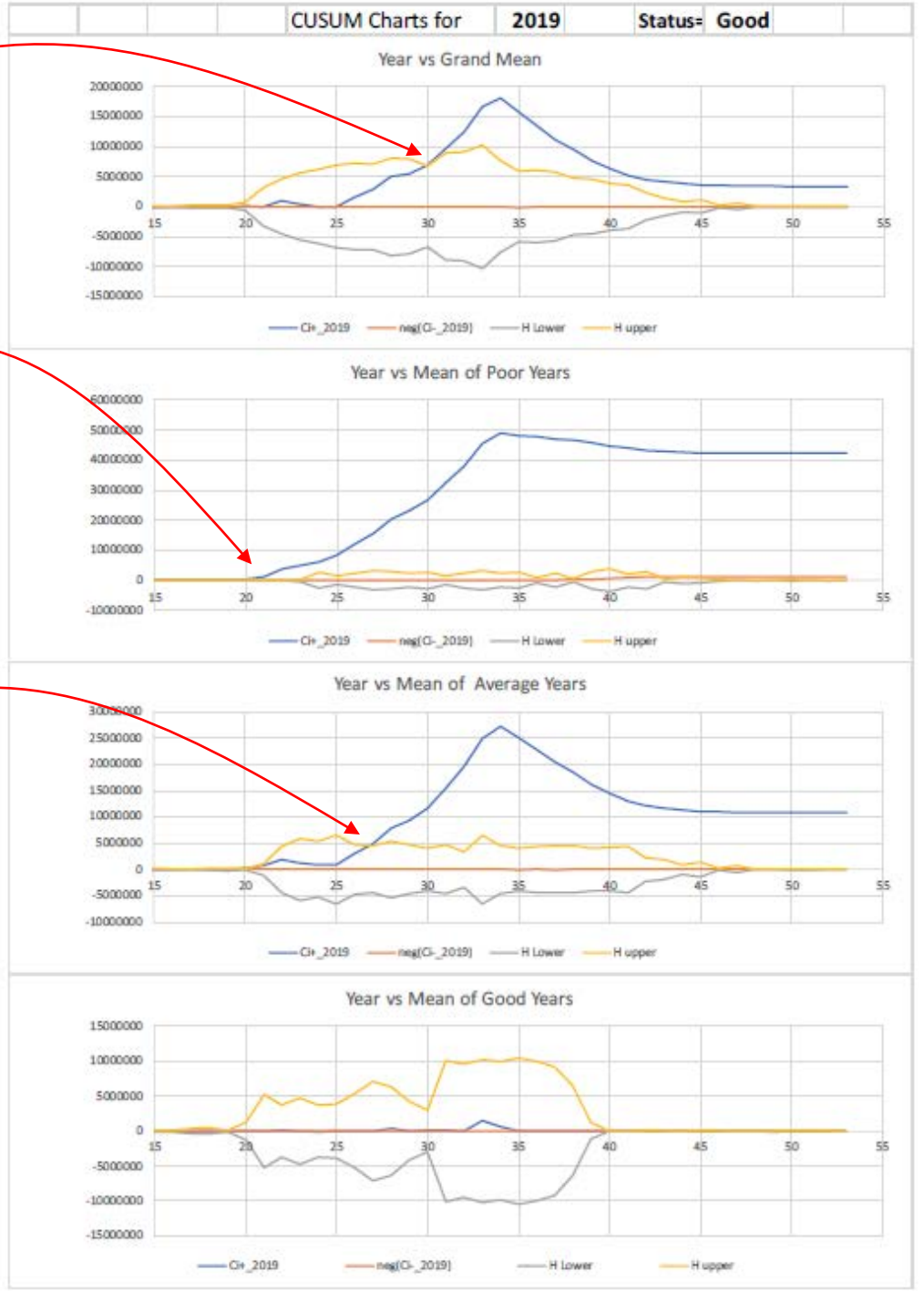
		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

Scoring a Poor Year 2016

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

Scoring the best year: 2019

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none



Scoring an average year: 2014

		First Out of Bounds Detection Year			
Year	Classification	All Years	Poor Years	Average Years	Good Years
1996	Ave	43+	20+	44+	28-
1997	Ave	none	27+	none	24-
1998	Good	30+	20+	25+	none
1999	Ave	none	30+	none	27-
2000	Ave	none	36+	none	27-
2001	Poor	none	none	none	24-
2002	Poor	none	none	none	24-
2003	Poor	none	43+	none	25-
2004	Good	38+	21+	28+	39+
2005	Ave	none	20+	none	28-
2006	Ave	none	27+	none	28-
2007	Ave	none	35+	none	24-
2008	Ave	42+	31+	45+	24-
2009	Ave	none	25+	none	28-
2010	Ave	none	21+	none	none
2011	Ave	none	20+	26+	39+
2012	Ave	none	33+	none	27-
2013	Poor	none	none	none	24-
2014	Ave	none	33+	none	27-
2015	Poor	none	none	none	24-
2016	Poor	45+	none	none	24-
2017	Good	38+	22+	32+	none
2018	Good	29+	21+	27+	none
2019	Good	31+	21+	27+	none

Sources of Uncertainty

- Basis for original classification of “Good”, “Average”, “Poor”
- Specification of seasonal means and variances based on samples
- Setting the control limits for a time varying mean and variance.