1

#### Implications of MAFMC Risk Policy for Multi-Year ABC Recommendations

### DRAFT

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Multi-year catch limits based on constant catches are often considered desirable by both managers and industry. The MidAtlantic Fishery Management Council has requested consideration of multi-year specifications based on average catches for a number of stocks. At the July 21-23, 2021 meeting of the SSC, two Council proposed average catch options for 2022 and 2023 could not be considered because the average catch policy resulted in an average ABC with P\* values above 0.5 in 2023. P\* is the probability of a given quota exceeding the overfishing threshold. Specifically, the 2023 quotas for scup and black sea bass, based on an average of the P\* estimates of ABC, resulted in P\*>0.5 in 2023.

It was suggested that this result is due in part to the Council's risk policy which allows P\* to be 0.49 when the B/Bmsy ratio exceeds 1.5. Compared to the previous risk policy, it allows for higher risk of overfishing for all levels of B/Bmsy ratios. As before, P\* is zero when B/Bmsy<0.1, but there are now two discontinuities at B/Bmsy=1 and 1.5 (Figure 1).

The P\* process for computation of ABCs over a multiyear period is iterative. The overfishing proxy Fmsy is applied to the current biomass to derive a catch defined as the Overfishing Limit (OFL). The OFL is adjusted downward to accommodate the uncertainty of the estimate by using the Coefficient of Variation (CV) derived by the SSC. Specifically, the derived OFL is assumed to be lognormally distributed with a mean given by the OFL from the assessment and a variance determined by the SSC's value of CV. The ABC is set to the level consistent with the Council's level of risk equal to P\* for the current value of B(t)/Bmsy. The resulting ABC is treated as a quota to update the population status. The Fmsy proxy is then applied to the updated population to estimate a new OFL for the next time step. The new OFL is then reduced to an ABC as described before.

The P\* approach has the desirable property that catches are consistent with the Council's risk policy at each time step. Does an average ABC derived from the P\* approach have the same property? From first principles one would not expect this to be true unless there was little or no variation in the ABC(t) estimates. Irrespective of slight violations of risk policy induced by an average ABC, the larger question is whether the average ABC results in P\*>0.5. This working paper addresses the implications of imposing an average catch based on the derived sequence of ABCs based on P\* estimates.





## **Operating Model**

None of the finfish stocks in the Northeast US use a parametric model for stock recruitment. Stochastic projections of future stock sizes in response to fishing mortality rates are typically modeled by randomly selecting recruitment levels from the empirical CDF from the assessment model. Although density independence is assumed, the consequence of this assumption is that maximum possible size of the future population, in the absence of fishing mortality, is determined by the average recruitment in the empirical CDF. Despite all the accounting for agespecific attributes of life history and fishery performance, the stock dynamics can be described by a linear mass balance equation:

$$B(t+1) = B(t) + f(B(t)) - Catch(t) - Loss(t)$$
(1)

Where B(t) =biomass at time t, f(B(t))=change in stock size as a result of change in average weight and prior recruitment, Catch(t) =landings + discards and Loss(t)= natural mortality.

The population equilibrium occurs when B(t+1)=B(t) or when f(B(t)) exactly balances total removals. Under an assumed constant average recruitment, f(B(t)) also becomes a constant.

For the purpose of illustration and to make things more interesting, Equation 1 can be generalized by assuming that f(B(t)) is proportional to stock size such that  $f(B(t))=\lambda B(t)$ . Using the catch equation, the Catch(t) and Loss(t) values can be written

Catch(t)=
$$F/Z$$
 (1-e<sup>-Z</sup>) B(t)  
=  $\alpha$  B(t)  
Loss(t)= $M/Z^*(1-e^{-Z})$  B(t)

 $=\beta B(t)$ 

Substituting these concepts into Equation 1 leads to

$$B(t+1) = (1+\lambda) B(t) - Catch(t) - Loss(t)$$
(2)

The P\* approach used by the Council and SSC modifies this process by decreasing  $\alpha$  in response to the risk of overfishing. The basis for this approach is summarized in the SSC documentation (2016, p 8)

"A central part the first three categories of ABC specification of the MAFMC ABC control rule is the determination of the uncertainty of the OFL. The MAFMC probabilistic approach begins with an estimate of the distribution of catch that can be taken when the population is fished at the fishing mortality threshold (FMT) given expected biomass when the catch limit will be implemented (OFL). The ABC is then determined by choosing the catch associated with a percentile (P\*) of the distribution, such that the ABC achieves a pre-specified probability of overfishing. The P\* represents the acceptable probability of overfishing, and the catch associated with a given percentile has a P\* probability of overfishing. In principle, this approach requires an accurate description of the OFL distribution. If the distribution of OFL is not accurate, the meaning of the P\* parameter is no longer the acceptable probability of overfishing – instead it simply is an ad hoc method for providing a buffer between ABC and OFL."

The ABC estimate based on P\*, denoted as  $C_{abc}(t)$  is computed by first finding the appropriate P\* from Figure 1 based on B(t)/Bmsy. The catch associated with application of Fmsy is the OFL and can be denoted as  $C_{ofl}(t)$ . The natural log of this estimate =  $\ln(C_{ofl}(t))$  serves as the mean of the log normal distribution function and the CV, determined by the SSC, defines the variance = $\ln(CV^2+1)$ . In words, this means find the  $C_{abc}(t)$  corresponding to the P\* percentile of a log normal distribution with mean =  $\ln(C_{ofl}(t))$  and variance =  $\ln(CV^2+1)$ . Or more mathematically,

$$C_{abc}(t) = \int_0^{P^*} LogNormal(x|\mu = \ln(C_{ofl}(t)), \sigma^2 = \ln(CV^2 + 1)) dx$$
(3)

The relationship between  $C_{abc}(t)$  and  $C_{ofl}(t)$ ) as defined by ratio B/Bmsy and the Council's risk policy (i.e., Figure 1) for varying levels of CV is shown in Figure 2. The effects of the change in risk policy at B/Bmsy =0.1, 1.0 and 1.5 are clearly evident.



Figure 2. Relationship between ratio ABC/OFL and B/Bmsy under the MAFMC's revised risk policy (Fig. 1) for OFL CV levels (60,100,150%) commonly used by the SSC.

## **MULTIYEAR PROJECTIONS BASED ON P\***

A multiyear projection using the P\* requires iterative application of the Council's risk policy. The steps in this iteration are

- 1. Compute  $C_{ofl}(t)$  based on B(t) and Fmsy
- 2. Update P\* given B(t)/Bmsy per Figure 1
- 3. Find  $C_{abc}(t)$  based on Equation 3.
- 4. Plug C<sub>abc</sub>(t) into the mass balance Equation 1
- 5. Update B(t+1) in response to the reduced value of  $C_{abc}(t)$
- 6. Compute  $C_{ofl}(t+1)$  based on B(t+1) and Fmsy
- 7. Go to step 2 and repeat.

To beat this dead horse further, the equation sequence for a T-year projection is

$$C_{ofl}(t) = \frac{F_{msy}}{Z} (1 - e^{-Z}) B_t$$
(4)

Update  $P^{*}(t)$  per Figure 1 for B(t)/Bmsy (5)

$$C_{abc}(t) = \int_{0}^{P^{*}(t)} LogNormal(\mu = \ln (C_{ofl}(t)), \sigma^{2} = \ln(CV^{2} + 1)) dx \quad (6)$$
$$L(t) = \frac{M}{Z} (1 - e^{-Z})B(t) \quad (7)$$
$$B(t+1) = B(t)(1+\lambda) - C_{abc}(t) - L(t) \quad (8)$$

$$C_{ofl}(t+1) = \frac{F_{msy}}{Z}(1-e^{-Z})B_{t+1}$$
(9)

Continue with Equation 5 until t=T.

#### MULTIYEAR PROJECTIONS BASED ON AVERAGE OF Cabc(t) for t=1,2, ..T

The average of the  $C_{abc}(t)$  has been endorsed by the Council as a reasonable balance between risk and desired aspects of fishery performance. The average,  $C_{abc\_avg}$  is computed as a simple average of the estimated  $C_{abc}(t)$ . The resulting P\* estimates derived using the updating process above will not be the same as the original P\*(t) unless the population is at equilibrium with no trend. If the stock is trending upward the application of  $C_{abc\_avg}$  will initially impede this growth because  $C_{abc\_avg}(t) > C_{abc}(t)$  but accelerate in later periods by reducing the catches, i.e.,  $C_{abc\_avg}(t+\Delta t) < C_{abc}(t+\Delta t)$ . If the stock is trending downward the reverse will be true. There is no guarantee however that you will get the same endpoint, B(T) given these different harvest scenarios, i.e., B(T|  $C_{abc\_avg}$ ) = B(T|  $C_{abc}(t)$ ). Moreover, Council policy allows for the relaxation of its risk policy under these circumstances such that the revised set of P\*'(t) given  $C_{abc\_avg}(t)$  can exceed the P\*(t) specified under the risk policy for B(t)/Bmsy. The underlying concept is the offsetting effects by paying on Tuesday for today's hamburger (Popeye, 1929, Wimpy op cit). However, under current understanding of Magnuson Stevens Act guidance, the upper bound on P\*(t) is a legal constraint equal to 0.5 as defined by US Court rulings. Hence any  $C_{abc\_avg}(t)$  that gives a P\*'(t)>0.5 is infeasible.

#### WHEN ARE MULTIYEAR PROJECTIONS BASED ON Cabc avg FEASIBLE?

The above equations allow for a simple parametric examination of the average ABC approach. Mike Wilberg's original spreadsheet for computation of ABCs and P\*s was modified to examine multiyear projections based on P\* and compare them to the realized P\* under the average of  $C_{abc}(t)$ . I also added a worksheet for finding an optimal average ABC that did not violate the P\*>0.5 criterion.

Here's a simple example based on an initial B(0)/Bmsy value of 2.0 and a value of  $1+\lambda=1.25$  and CV=60%. The P\* for this population is 0.49 or just under the legal limit of 0.5. In this scenario, the average ABC is computed as 123.13 and none of the P\*(t) are above 0.5. When this average

is substituted for the original  $C_{abc}(t)$  the P\* is 0.33 in year 1 since 123.13<154.94 in the P\* scenario. However, by year 3 the ABC = 123.31 gives a P\*=0.586. To facilitate comparison across multiple scenarios, the scenario is scored with respect to number of times the P\*>0.5 threshold was triggered and the last time period in which the violation occurred. In this example, the P\* criterion was triggered once in the third year

	P_star scenario		Aver	age Catch S	cenario			
Time	Biomass	ABC	Pstar	Biomass	ABC	Pstar	Scoring for P*>0.5	Rank Violation
0	500.00	154.94	0.490	500.00	123.13	0.330	0	0
1	395.60	121.74	0.490	427.41	123.13	0.438	0	0
2	313.86	92.71	0.470	347.49	123.13	0.586	1	3
3	252.87			259.49				
Average		123.129	0.483		123.129	0.451	1	3
							Scoring: n=# years P>0.5	
Model Parameters	Value							
М	0.2							
Fmsy	0.422							
Lambda	1.25							
B(0)/Bmsy	2							
CV	60%							

In the above scenario, the population biomass is decreasing from 500 to 252.9 because Fmsy +M is too large relative to 1+ $\lambda$ . From Eq. 1, the exact equilibrium occurs when  $(1+\lambda -\alpha -\beta)=1$ . This reduces to  $\lambda=(1-e^{-Z})$  or  $1+\lambda=2-e^{-Z}$ . This principle is shown in the following table where M=0.2, Fmsy=0.422 and  $1+\lambda = 1.46313$ .

	P_star scenario		Aver	age Catch S	cenario			
								Rank
Time	Biomass	ABC	Pstar	Biomass	ABC	Pstar	Scoring for P*>0.5	Violation
0	500.00	154.94	0.490	500.00	155.00	0.490	0	0
1	502.17	154.53	0.490	502.11	155.00	0.487	0	0
2	505.43	155.53	0.490	504.88	155.00	0.483	0	0
3	508.71			508.52				
Average		155.000	0.490		155.000	0.487	0	0
							Scoring: n=# years P>0.5	
Model Parameters	Value							
М	0.2							
Fmsy	0.422							
Lambda	1.46313038							
B(0)/Bmsy	2							
CV	60%							

It is now possible to examine the behavior of the average ABC for a wide range of parameters (Table 1). Results suggest that when the population is declining (i.e.,  $1+\lambda < 1.46313$ ) there is at most 1 violation of the P\*>0.5 criteria and that it occurs in the last year of the projection. When the population is exactly balanced (i.e.,  $1+\lambda = 1.46313$ ) no violations occur, as expected. When the population is growing (i.e.,  $1+\lambda > 1.46313$ ) there will be one or two violations. Single violations occur when B/Bmsy<1 and the violation occurs in the first year. At higher levels of population growth, violations occur in the first and second year of the projection. These conclusions are highly dependent on the parameters chosen, especially M and Fmsy but the

general principles can be established. The average ABC policy can induce violations of risk policy at all levels of B/Bmsy and population growth rates.

# Finding an Average ABC Based on P\* that Satisfies P\*(t)<0.5 for t=1,...T

The above analyses suggest that the average ABC approach can be problematic over a broad range of B/Bmsy and Population growth rate values. One way to eliminate this problem is pose it as a constrained optimization such that Copt<Cabc\_avg and P\*(t)<0.5 for t=1,2, ...T. The case below assumes the stock is at B/Bmsy=2 and declining with  $1+\lambda=1.25$ . The average catch of 123.13 creates a P\* violation in year 3.

	P_star scenario		Average Catch Scenario					
								Rank
Time	Biomass	ABC	Pstar	Biomass	ABC	Pstar	Scoring for P*>0.5	Violation
0	500.00	154.94	0.490	500.00	123.13	0.330	0	0
1	395.60	121.74	0.490	427.41	123.13	0.438	0	0
2	313.86	92.71	0.470	347.49	123.13	0.586	1	3
3	252.87			259.49				
Average		123.129	0.483		123.129	0.451	1	3
							Scoring: n=# years P>0.5	

The optimal solution, based upon minimizing (Copt<Cabc avg)<sup>2</sup> is

Average P*	0.483							
Average C_abc	123.13	<used a<="" in="" td=""><td>ve Catch So</td><td>enar</td><td>io</td><td></td><td></td><td></td></used>	ve Catch So	enar	io			
Opt ave C_abc	114.730327	<decision td="" v<=""><td>variable for</td><td>root</td><td>finder.</td><td></td><td></td><td></td></decision>	variable for	root	finder.			
(delta ave C)^2	70.5345531	<min td="" this="" v<=""><td>alue, subje</td><td>ct to</td><td>P*&lt;0.5 and</td><td>Opt<avera< td=""><td>ige ABC</td><td></td></avera<></td></min>	alue, subje	ct to	P*<0.5 and	Opt <avera< td=""><td>ige ABC</td><td></td></avera<>	ige ABC	
	P_	star scenario		Average Catch Scenario				
Time	Biomass	ABC	Pstar		Biomass	ABC	Pstar	Scoring for P*>0.5
Time 0	Biomass 500.00	ABC 154.94	Pstar 0.490		Biomass 500.00	ABC 114.73	Pstar 0.285	Scoring for P*>0.5 0
Time           0           1	Biomass 500.00 395.60	ABC 154.94 121.74	Pstar 0.490 0.490		Biomass 500.00 435.81	ABC 114.73 114.73	Pstar 0.285 0.375	Scoring for P*>0.5 0 0
Time           0           1           2	Biomass 500.00 395.60 313.86	ABC 154.94 121.74 92.71	Pstar 0.490 0.490 0.470		Biomass 500.00 435.81 365.13	ABC 114.73 114.73 114.73	Pstar 0.285 0.375 0.500	Scoring for P*>0.5 0 0 0
Time 0 1 2 3	Biomass 500.00 395.60 313.86 252.87	ABC 154.94 121.74 92.71	Pstar 0.490 0.490 0.470		Biomass 500.00 435.81 365.13 287.31	ABC 114.73 114.73 114.73	Pstar 0.285 0.375 0.500	Scoring for P*>0.5 0 0 0
Time           0           1           2           3           Average	Biomass 500.00 395.60 313.86 252.87	ABC 154.94 121.74 92.71 123.129	Pstar 0.490 0.490 0.470 0.483		Biomass 500.00 435.81 365.13 287.31	ABC 114.73 114.73 114.73 114.730	Pstar 0.285 0.375 0.500 0.387	Scoring for P*>0.5 0 0 0 0

The net loss in average catch is 123.13-114.73 or 6.8% each year. Note that the projected biomass at the end of the simulation is slightly higher 287.3 vs 252.87 due to the deferred catches. Interestingly, the biggest losses in yield are in the first year so this averaging policy might be less appealing.

## Discussion

Analyses suggest that performance of multiyear quotas depends not only on the ratio of current stock size to Bmsy but also the underlying trend in overall abundance. Rapid increases or decreases in stock size can increase the probability that an average ABC ( $C_{abc_avg}$ ) can lead to a P\*(t)>0.5 violation of policy. When the stock is declining, the violation will occur at the end of

the averaging period T. When the stock is increasing, the violation can occur in the middle or end of the averaging period. Given the simplicity of the operating model used in this exercise, more complex patterns of violations are certainly possible. Density-dependent process are not considered in these simulations, nor are effects of strong year classes, changes in average weight, maturation, and so forth. However, the model is sufficient for capturing the basic principles affecting the performance of average quotas over short periods. The optimization approach for finding a feasible multiyear quota could be approximated in an actual application by using a trial and error (or Newton Raphson) method for finding an average ABC that has  $P^*(t)<0.5$  for t=1,...,T.

# References

Scientific and Statistical Committee. 2016. Description and Foundation of the Mid-Atlantic Fishery Management Council's Acceptable Biological Catch Control Rule. Council web page link <u>https://www.mafmc.org/ssc</u>

Table 1. (A) Summary of number of times that an average ABC based on a P\*-based projection induces a P\* above 0.5 in a 3 year projection given F=0.422, M=0.2, CV=60%. Initial conditions for each scenario (B/Bmsy) define the row elements and the underlying rate of population growth (1+1) define the columns. Table elements indicate the number of times that the P\* exceeds 0.5. Table B shows the last year in which the P\* violation occurred.

	violation						
A	Score	decreasing}	<	1+ Lamb	da growth	rate>	{increasing
	1	0.9	1	1 25	1 46313	1 75	2
	0.25	0.5	0	0	0	0	1
	0.25	0	0	0	0	1	2
	0.5	0	0	0	0	2	2
	0.75	1	0	0	0	2	2
	0.83	1	0	0	0	2	2
	0.9	1	0	0	0	2	2
	0.95	1	0	0	0	2	2
	1	1	1	0	0	2	2
	1.05	1	1	0	0	2	2
	1.1	1	1	0	0	2	2
	1.15	1	1	0	0	2	2
	1.2	1	1	0	0	2	2
B/Bmsy	1.25	1	1	0	0	2	2
. ,	1.3	1	1	0	0	2	2
	1.35	1	1	1	0	2	2
	1.4	1	1	1	0	2	2
	1.45	1	1	1	0	2	2
	1.5	1	1	1	0	2	2
	1.55	1	1	1	0	2	2
	1.6	1	1	1	0	2	2
	1.65	1	1	1	0	2	2
	1.7	1	1	1	0	2	2
	1.75	1	1	1	0	2	2
	1.8	1	1	1	0	2	2
	2	1	1	1	0	2	2
	Max Rank						
В	Max Rank violation	decreasing}	<	1+ Lamb	da growth	rate>	{increasing
В	Max Rank violation	decreasing}	<	1+ Lambo 1.25	da growth	rate>	{increasing
В	Max Rank violation <u>3</u> 0 25	decreasing}	< 1 0	1+ Lambo 1.25	da growth 1.46313	rate> 1.75	{increasing,
В	Max Rank violation <u>3</u> 0.25	decreasing}	< 1 0	1+ Lambo 1.25 0	da growth 1.46313 0	rate> <u>1.75</u> 0	{increasing
В	Max Rank violation <u>3</u> 0.25 0.5	decreasing}	< 1 0 0	1+ Lambo 1.25 0 0	da growth 1.46313 0 0	rate> 1.75 0 1 2	{increasing
В	Max Rank violation <u>3</u> 0.25 0.5 0.75	decreasing}	< 1 0 0 0 0	1+ Lambo 1.25 0 0 0	da growth 1.46313 0 0 0 0 0	rate> 1.75 0 1 2 2	{increasing, 2 1 2 2 2
В	Max Rank violation <u>3</u> 0.25 0.5 0.75 0.85	<i>decreasing}</i> 0.9 0 0 0 3 2	< 1 0 0 0 0 0	1+ Lambo 1.25 0 0 0 0	da growth 1.46313 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2	<i>{increasing,</i> 2 1 2 2 2
В	Max Rank violation <u>3</u> 0.25 0.55 0.75 0.85 0.99	decreasing}	< 1 0 0 0 0 0 0 0	1+ Lambo 1.25 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2
В	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95	decreasing}	< 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1+ Lambo 1.25 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2	{increasing 2 1 2 2 2 2 2 2 2 2
В	Max Rank violation 3 0.25 0.5 0.5 0.5 0.85 0.9 0.95 1 1.05	decreasing}	< 1 0 0 0 0 0 0 0 0 0 3 3	1+ Lamba 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2
В	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05	decreasing}	< 1 0 0 0 0 0 0 3 3 3 2	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
В	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.11	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 3 3 3 3 2 2	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
В	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.1 1.15	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 3 3 3 3 3 2 2	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
B	Max Rank violation 3 0.25 0.5 0.75 0.85 0.95 0.95 1.1 1.05 1.11 1.15 1.2	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 3 3 3 3 3 3 0 0 0 0 0 0 0	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.99 0.95 1 1.05 1.1 1.15 1.2 1.25	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1.1 1.05 1.15 1.2 1.25 1.25 1.3	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1.05 1.15 1.15 1.2 1.25 1.3 1.35	decreasing} 0.9 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	<pre>&lt; 1 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3</pre>	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.15 1.25 1.25 1.3 1.35 1.4	decreasing} 0.9 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	1       0       0       0       0       0       3	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
<b>B</b> B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.15 1.15 1.25 1.3 1.35 1.4 1.45	decreasing} 0.9 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lamba 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.11 1.15 1.22 1.25 1.3 1.35 1.4 1.45 1.5	decreasing} 0.9 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lamba 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
<b>B</b> B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.12 1.25 1.33 1.35 1.4 1.45 1.55	decreasing} 0.9 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lamba 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing; 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.15 1.22 1.25 1.33 1.35 1.4 1.45 1.55 0.1.6	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
<b>B</b> B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.1 1.15 1.25 1.3 1.35 1.4 1.45 1.55 1.65	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	< 1 0 0 0 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.1 1.15 1.25 1.33 1.35 1.4 1.45 1.55 1.65 1.65	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	1         0 <tr< td=""><td>1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>rate&gt; 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td><td><pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre></td></tr<>	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.1 1.15 1.2 1.25 1.35 1.35 1.4 1.45 1.55 1.55 1.66	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	1         0         1         0 <tr< td=""><td>1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>rate&gt; 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td><td><pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre></td></tr<>	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>
B B/Bmsy	Max Rank violation 3 0.25 0.5 0.75 0.85 0.9 0.95 1 1.05 1.1 1.05 1.2 1.25 1.3 1.35 1.45 1.55 1.55 1.66 1.65 1.7 1.75 1.8	decreasing} 0.9 0 0 0 0 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	1         0         1         0 <tr< td=""><td>1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td><td>rate&gt; 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td><td><pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre></td></tr<>	1+ Lambo 1.25 0 0 0 0 0 0 0 0 0 0 0 0 0	da growth 1.46313 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	rate> 1.75 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	<pre>{increasing, 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</pre>